The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2021
Exam Number: 035582
Appendix: Formula sheets for 5 points
English Translation (3)

Mathematics

5 Points – 2nd Exam

Instructions for examinee

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

This exam consists of two parts, with a total of five questions.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions

Answer three questions of your choice
$-3\times33\frac{1}{3}=100$ points

ג. Material that may be used during the exam:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you use any draft paper outside the answer booklet your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תשפ"א, 2021
מספר השאלון: 035582
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות לנבחן

א. משך הבחינה: שעתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטריה אנליטית, וקטורים,
טריגונומטריה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות חזקה,
פונקציות מעריכיות ולוגריתמיות

עליך לענות על שלוש שאלות לבחירתך — $3\times33\frac{1}{3}=100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.
אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אל תעתיק את השאלה; סמן את מספרה בלבד.

(2) התחל כל שאלה בעמוד חדש.
רשום במחברת את שלבי הפתרון,
גם כאשר החישובים מתבצעים בעזרת מחשבון.
הסבר את כל פעולותיך, כולל חישובים,
בפירוט ובצורה ברורה מסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

כתוב במחברת הבחינה בלבד. רשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain <u>all</u> of your work, including calculations, clearly and in detail.
Lack of detail may lower your score or lead to the disqualification of your exam.

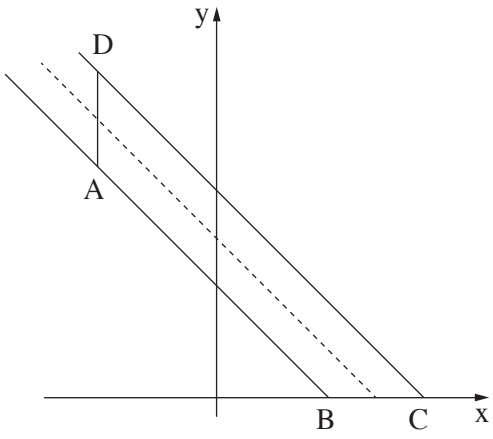
Answer <u>three</u> of the questions 1-5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. ABCD is a trapezoid $(AB \parallel DC)$.

Given: The distance between the trapezoid's bases, AB and DC, is $\sqrt{2}$, the midsegment of Trapezoid ABCD is located on the line $x + y - 4 = 0$.



א. Find the equations of the lines on which the bases of the trapezoid are located.

Given: The leg BC is located on the x-axis.

The canonical form of a parabola is $y^2 = 2px$ $(p > 0)$. The parabola is drawn such that the trapezoid vertices A and D are located on the parabola directrix, and the parabola focus is located on Vertex B or on Vertex C.

ב. **(1)** What is the equation of the parabola for which Trapezoid ABCD is the larger of the two possible trapezoids? Explain.

(2) What is the equation of the parabola for which Trapezoid ABCD is the smaller of the two possible trapezoids?

ג. A line is drawn parallel to the x-axis and also intersecting both of the parabolas that you found in Item **ב** at two points, E and F.

Find the equation of the locus on which the midpoints of the segments EF that have been formed in this way are located.

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2. ABC is a triangle.

Let: $\overrightarrow{AB}=\underline{u}$, $\overrightarrow{AC}=\underline{v}$.

Given: $A(0,2,-1)$, $B(-3,2,2)$,

the point $D(-2,3,1)$ is located on Segment BC such that $\overrightarrow{AD}=\frac{2}{3}\underline{u}+\frac{1}{3}\underline{v}$.

א. **(1)** Find the coordinates of Point C and prove that Triangle ABC is a right triangle.

(2) Find the equation of the plane ABC .

Point E is located on the plane ABC such that ABEC is a rectangle. Point M is the intersection point of the diagonals [מפגש האלכסונים] in this rectangle.

S is a point such that MS is perpendicular to the plane ABEC .

ב. **(1)** Find a parametric representation for the line MS ; explain why, for any such point S , SABEC is a right pyramid.

(2) Give an example of coordinates of a point S such as described in Sub-item **ב(1)**. Calculate the angle SAB for the point S that you found.

(3) For the point S that you found, is there an additional point, P , such that PABEC is a right pyramid for which $\sphericalangle SAB=\sphericalangle PAB$?

If there is such a point P , find its coordinates. If not, explain.

3. Given: The equation $i\cdot z^6=\frac{1}{64}$ (z is a complex number).

א. Find all the solutions to the given equation.

The solutions of the given equation correspond to the vertices of a convex polygon on a Gauss plain.

ב. Show that for each of the polygon's vertices there is exactly one vertex, such that the line that connects them passes through the origin.

Each of the solutions of the given equation is multiplied by a constant complex number, w .

ג. Explain why the sum of the numbers that was obtained is zero.

Given: $w=\frac{\sqrt{3}}{2}+\frac{1}{2}i$.

ד. Write an equation whose solutions are the 12 numbers—the solutions to the equation given at the beginning of this question and the numbers obtained by multiplying these solutions by w .

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Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Given: the function $f(x)=\frac{-4}{e^{2x}-4e^{x}+3}$.

א. **(1)** Find the domain [תחום ההגדרה] of the function $f(x)$.

(2) Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes.

(3) Find the coordinates of the extrema [נקודות קיצון] of the function $f(x)$ and determine what type of extrema they are (if there are any).

(4) Find the intervals on which the function $f(x)$ is increasing and the intervals on which it is decreasing.

(5) Sketch a graph of the function $f(x)$.

ב. Explain why $\int_{b-3}^{b} f(x)\,dx < -4$ is valid for any $b<0$.

The given function $g(x)=\frac{k}{f(x)}$ is defined on the same domain as the function $f(x)$.
k is a parameter.

Given: Function $g(x)$ has a minimum.

ג. What is the possible range of values for k ? Explain.

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5. Given: the function $f(x)=\frac{1}{(\ell n(x))^3-1}+1$.

א. **(1)** Find the domain [תחום ההגדרה] of the function $f(x)$.

(2) Find the equations of the asymptotes of the function $f(x)$ that are perpendicular to the axes.

(3) Find the intervals on which the function $f(x)$ is increasing and on which it is decreasing (if there are any).

(4) Find the coordinates of the intersection points of the graph of the function $f(x)$ with the axes (if there are any).

(5) Sketch a graph of the function $f(x)$.

ב. The line $y=k$ does not intersect the graph of the function $f(x)$ (k is a parameter).

Find k . Explain.

ג. We define $T(x)=\int_{e^{-1}}^{x} f(x)\,dx \quad e^{-1}\le x<e$.

(1) Consider the three values of x below (I-III). For which of these is the value of $T(x)$ the greatest? Explain.

I) $x=\frac{1}{2}$ II) $x=1$ III) $x=2$

(2) Explain why $T(x)<1$ is valid for any $e^{-1}\le x<e$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך