Mathematics Formula Sheets

5 points

New Program

Algebra

$(a \pm b)^2 = a^2 \pm 2ab + b^2$ $\qquad$ $a^2 - b^2 = (a-b)(a+b)$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$ $\qquad$ $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Quadratic equation: $ax^2 + bx + c = 0 \ (a \neq 0)$ $\qquad$ Roots: $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sequences:

	Arithmetic sequence	**Geometric sequence**
Recurrence relation:	$\begin{cases} a_1 = a \\ a_{n+1} = a_n + d \end{cases}$	$\begin{cases} a_1 = a \\ a_{n+1} = a_n \cdot q \end{cases}$
*n*th term:	$a_n = a_1 + (n-1)d$	$a_n = a_1 \cdot q^{n-1}$
Sum:	$S_n = \dfrac{n(a_1 + a_n)}{2}$ $S_n = \dfrac{n\,[2a_1 + (n-1)d]}{2}$	$S_n = \dfrac{a_1(q^n - 1)}{q - 1}$ $\quad q \neq 1$ Sum of an infinite sequence with convergent sum: $S = \dfrac{a_1}{1-q}$

Exponents: $(a \neq 0\ ,\ b \neq 0)$

$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$	$(a^x)^y = a^{x \cdot y}$	$\dfrac{a^x}{a^y} = a^{x-y}$	$a^x \cdot a^y = a^{x+y}$
$(a \cdot b)^x = a^x \cdot b^x$	$a^{\frac{m}{n}} = \sqrt[n]{a^m}\ , a > 0$	$a^{-x} = \dfrac{1}{a^x}$	

Logarithms (by domain constraints):

$\log_a(a^b) = b$	$a^{\log_a x} = x$	$\log_a(x^b) = b \cdot \log_a x$
$\log_a x + \log_a y = \log_a(x \cdot y)$	$\log_a x - \log_a y = \log_a\left(\dfrac{x}{y}\right)$	$\log_c x = \dfrac{\log_a x}{\log_a c}$

Growth and decay: Quantity after t units of time: $f(t) = f(0) \cdot q^t$, where q is the growth/decay coefficient per unit of time t

Probability

Conditional probability: $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

Bernoulli formula: Probability of k successes out of n trials in a binomial distribution, where the probability of success is p :

$$P_n(k) = \binom{n}{k} p^k \cdot (1-p)^{n-k} \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Trigonometry and Geometry

Identities:

$\sin(-\alpha) = -\sin\alpha$

$\cos(-\alpha) = \cos\alpha$

$\sin(\pi - \alpha) = \sin\alpha$

$\cos(\pi - \alpha) = -\cos\alpha$

$\sin(\frac{\pi}{2} - \alpha) = \cos\alpha$

$\cos(\frac{\pi}{2} - \alpha) = \sin\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$

$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$

$\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$

$\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$

$\sin\alpha - \sin\beta = 2\sin\frac{\alpha - \beta}{2} \cdot \cos\frac{\alpha + \beta}{2}$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha + \beta}{2} \cdot \cos\frac{\alpha - \beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\frac{\alpha + \beta}{2} \cdot \sin\frac{\alpha - \beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2} \cdot \cos\frac{\alpha - \beta}{2}$

Law of sines: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$ (R – radius of the circle that circumscribes the triangle)

Law of cosines: $c^2 = a^2 + b^2 - 2a \cdot b \cdot \cos\gamma$ (γ – angle between the sides a and b)

Area of a triangle: $S = \frac{1}{2} \cdot b \cdot c \cdot \sin\alpha$ (α – angle between the sides b and c)

$S = \frac{a^2 \cdot \sin\beta \cdot \sin\gamma}{2 \cdot \sin(\beta + \gamma)}$ (β and γ – angles adjacent to the side a)

Area of a circle: $S = \pi \cdot R^2$ **Perimeter of a circle**: $P = 2\pi \cdot R$ (R – radius)

Length of an arc of α radians: $\ell = \alpha \cdot R$ **Area of a sector** of α radians: $S = \frac{1}{2}\alpha \cdot R^2$ (R – radius)

Differential and Integral Calculus

Derivatives:

$(\frac{1}{x})' = -\frac{1}{x^2}$	$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	$(x^t)' = t \cdot x^{t-1}$ (t is real)
$(\sin x)' = \cos x$	$(\cos x)' = -\sin x$	$(\tan x)' = \frac{1}{\cos^2 x}$
$(a^x)' = a^x \cdot \ell na$	$(\ell og_a x)' = \frac{1}{x \cdot \ell na}$	

Derivative of the product of functions: $[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Derivative of the quotient of functions: $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Derivative of a composite function: $[f(u(x))]' = f'(u) \cdot u'(x)$

$u'(x)$ is the derivative of u by x (inner derivative)

and $f'(u)$ is the derivative of f by u (outer derivative)

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Integrals:

$\int x^t dx = \frac{x^{t+1}}{t+1} + C$ (t is real, $t \neq -1$)	$\int \cos x dx = \sin x + C$	$\int \sin x dx = -\cos x + C$	$\int \frac{1}{\cos^2 x} dx = \tan x + C$
$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$	$\int e^x dx = e^x + C$	$\int a^x dx = \frac{a^x}{\ell na} + C$	$\int \frac{1}{x} dx = \ell n\lvert x \rvert + C$

If $F(x)$ is a primitive function of $f(x)$, then: $\int f(mx+b)dx = \frac{1}{m}F(mx+b) + C \quad (m \neq 0)$

$$\int f[u(x)] \cdot u'(x)dx = F[u(x)] + C$$

Analytical Geometry

Line:

Slope m of a line that passes through the points (x_1, y_1) and (x_2, y_2): $m = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_1 \neq x_2)$

Equation of a line of slope m, that passes through the point (x_1, y_1): $y - y_1 = m(x - x_1)$

Coordinates of Point C, which (internally) divides the segment whose end points are $A(x_1, y_1)$, $B(x_2, y_2)$, at the ratio $\frac{AC}{BC} = \frac{k}{\ell}$: $\left(\frac{\ell x_1 + kx_2}{k+\ell}, \frac{\ell y_1 + ky_2}{k+\ell}\right)$

Two lines of slopes m_1 m_2 are perpendicular to each other if and only if: $m_1 \cdot m_2 = -1$

Distance d between the points $A(x_1, y_1)$ and $B(x_2, y_2)$: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Distance d between the point (x_0, y_0) and the line $Ax + By + C = 0$: $d = \frac{\lvert Ax_0 + By_0 + C \rvert}{\sqrt{A^2 + B^2}}$

Distance d between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$: $d = \frac{\lvert C_1 - C_2 \rvert}{\sqrt{A^2 + B^2}}$

Circle:

Equation of the tangent to a circle $(x-a)^2 + (y-b)^2 = R^2$ at the point (x_0, y_0) on the circle: $(x_0 - a) \cdot (x - a) + (y_0 - b) \cdot (y - b) = R^2$

Parabola:

Equation of the tangent to a parabola $y^2 = 2px$ at the point (x_0, y_0) on the parabola: $y \cdot y_0 = p(x + x_0)$

Directrix of a parabola whose equation is $y^2 = 2px$: $x = -\frac{p}{2}$

Focus of a parabola whose equation is $y^2 = 2px$: $F\left(\frac{p}{2}, 0\right)$

	Equation	Distance of focus from origin	
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$c = \sqrt{a^2 - b^2}$	
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$c = \sqrt{a^2 + b^2}$	Equations of hyperbola asymptotes: $y = \pm \frac{b}{a}x$

Complex Numbers

Polar representation of the product of the complex numbers $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$:

$$z_1 \cdot z_2 = r_1 \cdot r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

De Moivre's Theorem: $[R(\cos\varphi + i\sin\varphi)]^n = R^n(\cos n\varphi + i\sin n\varphi)$

Solutions to the equation $z^n = R(\cos\varphi + i\sin\varphi)$:

$$z_k = \sqrt[n]{R}\left[\cos\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right)\right]$$

$k = 0, 1, 2, ..., n-1$

Three-Dimensional Objects

Prism and cylinder:	Volume:	$V = B \cdot h$	(B – base area, h – object height)
Pyramid and cone:	Volume:	$V = \frac{B \cdot h}{3}$	(B – base area, h – object height)
Right circular cone:	Lateral surface area:	$M = \pi R \ell$	(R – base radius, ℓ – generatrix/slant edge)
Sphere:	Volume:	$V = \frac{4}{3}\pi R^3$	(R – sphere radius)
	Surface area:	$M = 4\pi R^2$	(R – sphere radius)

Vectors

Length of a vector $\underline{u} = (u_1, u_2, u_3)$: $|\underline{u}| = \sqrt{\underline{u} \cdot \underline{u}} = \sqrt{u_1^2 + u_2^2 + u_3^2}$

Scalar product of two vectors $\underline{u} = (u_1, u_2, u_3)$ and $\underline{v} = (v_1, v_2, v_3)$: $\underline{u} \cdot \underline{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + u_3 \cdot v_3$

Scalar product where α is the angle between the vectors $\underline{u}$ and $\underline{v}$: $\underline{u} \cdot \underline{v} = |\underline{u}| \cdot |\underline{v}| \cdot \cos\alpha$

Distance of the point $P(x_1, y_1, z_1)$ from the plane $ax + by + cz + d = 0$:

$$\frac{|a \cdot x_1 + b \cdot y_1 + c \cdot z_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Distance between parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$:

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

Angle β between the line $\underline{x} = \underline{v} + t \cdot \underline{u}$ and the plane $ax + by + cz + d = 0$, where $\underline{n} = (a, b, c)$:

$$\sin\beta = \frac{|\underline{n} \cdot \underline{u}|}{|\underline{n}| \cdot |\underline{u}|}$$

Angle α between the planes

$$\left.\begin{array}{l} a_1x + b_1y + c_1z + d_1 = 0 \text{ where } \underline{n_1} = (a_1, b_1, c_1) \\ a_2x + b_2y + c_2z + d_2 = 0 \text{ where } \underline{n_2} = (a_2, b_2, c_2) \end{array}\right\} : \cos\alpha = \frac{|\underline{n_1} \cdot \underline{n_2}|}{|\underline{n_1}| \cdot |\underline{n_2}|}$$