Mathematics Formula Sheets

4 Points

New Program

Algebra

$$(a \pm b)^2 = a^2 \pm 2ab + b^2 \qquad a^2 - b^2 = (a - b)(a + b)$$

Quadratic equation: $ax^2 + bx + c = 0 \quad (a \neq 0)$ Roots: $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sequences:

	Arithmetic sequence	Geometric sequence
Recurrence relation:	$\begin{cases} a_1 = a \\ a_{n+1} = a_n + d \end{cases}$	$\begin{cases} a_1 = a \\ a_{n+1} = a_n \cdot q \end{cases}$
*n*th term:	$a_n = a_1 + (n-1)d$	$a_n = a_1 \cdot q^{n-1}$
Sum:	$S_n = \frac{n(a_1 + a_n)}{2}$ $S_n = \frac{n[2a_1 + (n-1)d]}{2}$	$S_n = \frac{a_1(q^n - 1)}{q - 1} \quad q \neq 1$ Sum of an infinite sequence with convergent sum: $S = \frac{a_1}{1 - q}$

Exponents: $(a \neq 0\ ,\ b \neq 0)$

$a^x \cdot a^y = a^{x+y}$	$\frac{a^x}{a^y} = a^{x-y}$	$(a^x)^y = a^{x \cdot y}$	$(a \cdot b)^x = a^x \cdot b^x$	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	$a^{-x} = \frac{1}{a^x}$

Growth and decay: quantity after t units of time: $f(t) = f(0) \cdot q^t$, where q is the growth/decay coefficient per unit of time t

$q = 1 \pm \frac{p}{100}$ (p – percentage of growth/decay per unit of time)

Logarithms (by domain constraints):

$a^{\log_a x} = x$	$\log_a(a^b) = b$	$a^b = x$ equivalent to $\log_a x = b$
$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$	$\log_a x + \log_a y = \log_a(x \cdot y)$	$\log_a(x^b) = b \cdot \log_a x$

Geometry

The slope m of a line that passes through the points (x_1, y_1) and (x_2, y_2) and is not perpendicular to the x-axis: $m = \frac{y_2 - y_1}{x_2 - x_1}$

The equation of a line of slope m that passes through the point (x_1, y_1): $y - y_1 = m(x - x_1)$

The coordinates of the midpoint $M(x_M, y_M)$ of a segment whose extreme points are $A(x_1, y_1)$ and $B(x_2, y_2)$: $x_M = \frac{x_1 + x_2}{2}$, $y_M = \frac{y_1 + y_2}{2}$

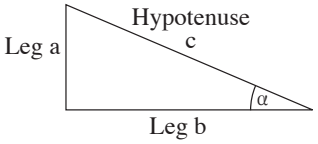
The distance d between the points $A(x_1, y_1)$ and $B(x_2, y_2)$: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Two lines of slopes m_1 and m_2 are perpendicular to each other if and only if: $m_1 \cdot m_2 = -1$

The equation of a circle of center (a, b) and radius R: $(x - a)^2 + (y - b)^2 = R^2$

The acute angle α between a line of the form $y = mx + b$ and the x-axis: $\tan\alpha = |m|$

Trigonometric functions in a right triangle:



$\sin\alpha = \frac{a}{c}$, $\cos\alpha = \frac{b}{c}$, $\tan\alpha = \frac{a}{b}$

Pythagoras' theorem: $a^2 + b^2 = c^2$

Trigonometric identities:

$\sin(90^o - \alpha) = \cos\alpha$	$\cos(90^o - \alpha) = \sin\alpha$	$\sin(180^o - \alpha) = \sin\alpha$	$\cos(180^o - \alpha) = -\cos\alpha$

Law of sines: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$ (R – radius of the circle that circumscribes the triangle)

Plane shapes:

Area of a triangle: $S = \frac{1}{2} \cdot b \cdot c \cdot \sin\alpha$ (α – angle between the sides b and c)

Area of a parallelogram: $S = a \cdot h$ (h – height to the side a)

Area of a trapezoid: $S = \frac{(a + b) \cdot h}{2}$ (a, b – bases of the trapezoid, h – height)

Area of a circle: $S = \pi \cdot R^2$ Circumference of a circle: $P = 2\pi \cdot R$ (R – radius)

Three-dimensional objects:

Volume of a prism: $V = B \cdot h$ Volume of a pyramid: $V = \frac{B \cdot h}{3}$ (B – base area, h – object height)

Vectors:

Given the vectors $\underline{u} = (u_1, u_2, u_3)$ and $\underline{v} = (v_1, v_2, v_3)$:

Length of vector: $|\underline{u}| = \sqrt{\underline{u} \cdot \underline{u}}$ $|\underline{u}| = \sqrt{(u_1)^2 + (u_2)^2 + (u_3)^2}$

Scalar product: $\underline{u} \cdot \underline{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + u_3 \cdot v_3$ $\underline{u} \cdot \underline{v} = |\underline{u}| \cdot |\underline{v}| \cdot \cos\alpha$ (α – angle between vectors $\underline{u}$, $\underline{v}$)

Angle α between the vectors $\underline{u}$, $\underline{v}$: $\cos\alpha = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}| \cdot |\underline{v}|}$

Differential and Integral Calculus

Derivatives:

$(x^t)' = t \cdot x^{t-1}$ (t is a real number)	$\left(\frac{a}{x}\right)' = -\frac{a}{x^2}$	$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	$(e^x)' = e^x$	$(\ell nx)' = \frac{1}{x}$
$[(f(x))^t]' = t \cdot [f(x)]^{t-1} \cdot f'(x)$ (t is a real number)	$\left[\frac{1}{f(x)}\right]' = -\frac{f'(x)}{[f(x)]^2}$	$[\sqrt{f(x)}]' = \frac{f'(x)}{2\sqrt{f(x)}}$	$[e^{f(x)}]' = e^{f(x)} \cdot f'(x)$	$[\ell n(f(x))]' = \frac{f'(x)}{f(x)}$

Derivative of the product of functions: $[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Derivative of the quotient of functions: $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Derivative of a complex function: $[f(u(x))]' = f'(u) \cdot u'(x)$

$u'(x)$ is the derivative of u by x (interior derivative)

and $f'(u)$ is the derivative of f by u (exterior derivative)

Integrals $(m \neq 0)$:

$\int x^t dx = \frac{x^{t+1}}{t+1} + C$ (t is a real number, $t \neq -1$)	$\int (mx+b)^t dx = \frac{(mx+b)^{t+1}}{m \cdot (t+1)} + C$ (t is a real number, $t \neq -1$)
$\int \frac{a}{x^2} dx = -\frac{a}{x} + C$	$\int \frac{a}{(mx+b)^2} dx = \frac{-a}{m \cdot (mx+b)} + C$
$\int e^x dx = e^x + C$	$\int e^{mx+b} dx = \frac{e^{mx+b}}{m} + C$
$\int \frac{1}{x} dx = \ell n\lvert x \rvert + C$	$\int \frac{1}{mx+b} dx = \frac{\ell n\lvert mx+b \rvert}{m} + C$

Probability

Conditional probability: $P(A/B) = \frac{P(A \cap B)}{P(B)}$

Probability of A and B , where A and B are independent events: $P(A \cap B) = P(A) \cdot P(B)$

Statistics

$f_1, f_2, \ldots, f_n$ are the frequencies of $x_1, x_2, \ldots, x_n$ respectively, and $N = f_1 + f_2 + \ldots + f_n$.

Average: $\bar{x} = \frac{x_1 f_1 + x_2 f_2 + \ldots + x_n f_n}{N}$

Standard deviation: $S = \sqrt{\frac{(x_1 - \bar{x})^2 \cdot f_1 + (x_2 - \bar{x})^2 \cdot f_2 + \ldots + (x_n - \bar{x})^2 \cdot f_n}{N}}$

Correlation coefficient r : $r = \frac{1}{N \cdot S_x \cdot S_y}[(x_1 - \bar{x})(y_1 - \bar{y}) + \ldots + (x_N - \bar{x})(y_N - \bar{y})]$

$r = \frac{1}{N}[(z_x)_1(z_y)_1 + \ldots + (z_x)_N(z_y)_N]$

Slope m of a regression line: $m = r \cdot \frac{S_y}{S_x}$ Regression line equation: $y - \bar{y} = m(x - \bar{x})$

Standard score: $z = \frac{x - \bar{x}}{S}$

Normal distribution:

$P(z > z_x) = P(z < -z_x)$	$P(z > z_x) = 1 - P(z < z_x)$	$P(z_1 < z < z_2) = P(z < z_2) - P(z < z_1)$

Normal distribution table

z	0	1	2	3	4	5	6	7	8	9
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0046	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0135	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0227	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0238	0.0233
-1.8	0.0359	0.0350	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0550	0.0540	0.0530	0.0520	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0670	0.0650	0.0640	0.0630	0.0620	0.0610	0.0590	0.0580	0.0570	0.0560
-1.4	0.0810	0.0790	0.0780	0.0760	0.0750	0.0740	0.0720	0.0710	0.0690	0.0680
-1.3	0.0970	0.0950	0.0930	0.0920	0.0900	0.0890	0.0870	0.0850	0.0840	0.0820
-1.2	0.1150	0.1130	0.1110	0.1090	0.1070	0.1060	0.1040	0.1020	0.1000	0.0980
-1.1	0.1360	0.1340	0.1310	0.1290	0.1270	0.1250	0.1230	0.1210	0.1190	0.1170
-1.0	0.1590	0.1560	0.1540	0.1520	0.1490	0.1470	0.1450	0.1420	0.1400	0.1380
-0.9	0.1840	0.1810	0.1790	0.1760	0.1740	0.1710	0.1680	0.1660	0.1630	0.1610
-0.8	0.2120	0.2090	0.2060	0.2030	0.2000	0.1980	0.1950	0.1920	0.1890	0.1870
-0.7	0.2420	0.2390	0.2360	0.2330	0.2300	0.2270	0.2240	0.2210	0.2180	0.2150
-0.6	0.2740	0.2710	0.2680	0.2640	0.2610	0.2580	0.2550	0.2510	0.2480	0.2450
-0.5	0.3080	0.3050	0.3010	0.2980	0.2950	0.2910	0.2880	0.2840	0.2810	0.2780
-0.4	0.3450	0.3410	0.3370	0.3340	0.3300	0.3260	0.3230	0.3190	0.3160	0.3120
-0.3	0.3820	0.3780	0.3750	0.3710	0.3670	0.3630	0.3590	0.3560	0.3520	0.3480
-0.2	0.4210	0.4170	0.4130	0.4090	0.4050	0.4010	0.3970	0.3940	0.3900	0.3860
-0.1	0.4600	0.4560	0.4520	0.4480	0.4440	0.4400	0.4360	0.4320	0.4290	0.4250
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4800	0.4760	0.4720	0.4680	0.4640
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5200	0.5240	0.5280	0.5320	0.5360
0.1	0.5400	0.5440	0.5480	0.5520	0.5560	0.5600	0.5640	0.5680	0.5710	0.5750
0.2	0.5790	0.5830	0.5870	0.5910	0.5950	0.5990	0.6030	0.6060	0.6100	0.6140
0.3	0.6180	0.6220	0.6250	0.6290	0.6330	0.6370	0.6410	0.6440	0.6480	0.6520
0.4	0.6550	0.6590	0.6630	0.6660	0.6700	0.6740	0.6770	0.6810	0.6840	0.6880
0.5	0.6920	0.6950	0.6990	0.7020	0.7050	0.7090	0.7120	0.7160	0.7190	0.7220
0.6	0.7260	0.7290	0.7320	0.7360	0.7390	0.7420	0.7450	0.7490	0.7520	0.7550
0.7	0.7580	0.7610	0.7640	0.7670	0.7700	0.7730	0.7760	0.7790	0.7820	0.7850
0.8	0.7880	0.7910	0.7940	0.7970	0.8000	0.8020	0.8050	0.8080	0.8110	0.8130
0.9	0.8160	0.8190	0.8210	0.8240	0.8260	0.8290	0.8320	0.8340	0.8370	0.8390
1.0	0.8410	0.8440	0.8460	0.8480	0.8510	0.8530	0.8550	0.8580	0.8600	0.8620
1.1	0.8640	0.8660	0.8690	0.8710	0.8730	0.8750	0.8770	0.8790	0.8810	0.8830
1.2	0.8850	0.8870	0.8890	0.8910	0.8930	0.8940	0.8960	0.8980	0.9000	0.9020
1.3	0.9030	0.9050	0.9070	0.9080	0.9100	0.9110	0.9130	0.9150	0.9160	0.9180
1.4	0.9190	0.9210	0.9220	0.9240	0.9250	0.9260	0.9280	0.9290	0.9310	0.9320
1.5	0.9330	0.9350	0.9360	0.9370	0.9380	0.9390	0.9410	0.9420	0.9430	0.9440
1.6	0.9450	0.9460	0.9470	0.9480	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9650	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9954	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
z	0	1	2	3	4	5	6	7	8	9