Integrating Mathematical Applets in the Teaching Sequence

Abstract

Integrating digital technologies in mathematics education has increasingly become an established practice over the last few decades. Specifically, engaging students with mathematical applets (MAPPs) enhances learning. It is widely agreed that in order to achieve a successful integration, learning and teaching need to be well-designed for the use of digital technologies; the teacher should orchestrate learning in an effective way; and the digital technologies should be naturally, coherently used in everyday educational contexts. However, despite growing evidence regarding the contribution of MAPPs to mathematics education, the level of their adoption into common teaching practices remains quite low. Therefore, the proposed research goal is twofold: 1) to promote a meaningful, long-lasting, pedagogical change among mathematics teachers through the systematic and structured integration of MAPPs as an important resource in their teaching sequence; and, as a result, 2) to enhance students' learning progress by using MAPPs. This goal will be achieved by equipping teachers with empirically based practices and data-driven tools to support their decision-making regarding the incorporation of MAPPs in class and across the curriculum.

The proposed research is comprised of four main components that implement different methodologies. Developing a framework for choosing MAPPs for teaching will be based on the exploration of existing repositories of MAPPs and on interviews with teachers. The professional development program will be developed, applied, evaluated, and modified, based on multiple case studies during the three years of research, which will include classroom observations and teacher interviews. Developing data-driven decision-support tools for teachers will take a learning analytics approach, analyzing log files of students' interactions with the MAPPs. Assessing students' learning will be accomplished with log-based analyses of students' interactions with the system, as well as by external, validated national assessment (the Meitzav examination) – both will be compared with control groups. The proposed research has both practical and theoretical implications, which take pedagogical, cognitive, and affective aspects into account. We identify four important implications:

1. Characterizing ways to integrate MAPPs in mathematics education in a well-informed manner, while encouraging mathematical thinking and increasing students' interest.
2. Empowering mathematics teachers as leaders, by supporting them with practices and tools that will assist them to navigate through the wide range of available digital resources, to wisely choose those that best fit their educational agenda, and to effectively use them.
3. Creating a professional development program model to be used by mathematics teachers that will specifically address the orchestration of available digital resources in the teaching sequence.
4. Applying learning analytics methods based on log file analysis, to extract important insights from students' interactions with the digital environment.

Integrating Mathematical Applets in the Teaching Sequence

RESERCH PROGRAM

Background, Goals, and Research Questions

Integrating digital technologies in mathematics education has increasingly become an established practice over the last few decades. However, many questions still arise regarding this integration and the role of digital technologies in mathematics education is still being studied. It is widely agreed that in order to achieve a successful integration, learning and teaching should be well-designed for the use of digital technologies; the teacher should orchestrate learning in an effective way (Díaz, Nussbaum, & Varela, 2015); and the digital technologies should be naturally and coherently used in everyday educational contexts (Drijvers, 2015; Lawless & Pellegrino, 2007). Hence, the focus of the proposed research is on exploring the integration of digital technologies in mathematics education, as well as on empirically developing a set of practices and data-driven decision-supporting tools for teachers, to assist them in designing and orchestrating the use of digital technologies in the mathematics classroom in a way that will facilitate a systematic and structured whole.

More specifically, the proposed research will explore the integration of mathematical applets (MAPPs) in the classroom, across the curriculum. Applets are software components that are designed to perform specific, designated services. Because of their small size and ease of use, educational applets have become very popular. In mathematics education in particular, MAPPs have been used extensively for a wide range of purposes such as demonstrating concepts or solution methods, explaining mathematical ideas, practicing, and generating examples and data (Boyle et al., 2016).

In recent years, teachers have been presented with large repositories (either free or commercial) of digital, educational resources from which to select materials to be incorporated into their instruction. For example, Illumination, a project designed by the National Council of Teachers of Mathematics in the US, offers dozens of applets of different types[1]; Wolfram, a leading scientific website, offers thousands of interactive MAPPs[2]; KHAN academy offers thousands of short clips and interactive applets[3]; Matific, a commercial site for teaching mathematics, is based solely on MAPPs and has a few hundred of them[4]; and many other independent websites offer dozens and hundreds of applets for various grade levels in many mathematical domains (e.g., CoolMath-Games.com, ABCYa.com, LearningGamesForKids.com[5]). These online repositories supply teachers who wish to use digital resources in their classrooms. However, despite the growing availability of digital educational

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[1] https://illuminations.nctm.org [accessed July 2016].
[2] Wolfram Demonstrations Project, http://demonstrations.wolfram.com [accessed July 2016].
[3] https://www.khanacademy.org/math, [accessed July 2016]
[4] http://www.matific.com [accessed July 2016].
[5] All sites mentioned were accessed July 2016.

resources in general, and in mathematics in particular, along with the many advantages inherent in them, the level of MAPPs' adoption into common teaching practices remains quite low (Joubert, 2013; Recker et al., 2005). These ever-growing repositories raise some new concerns and challenges for teachers, as they need to be aware of, search for, assess, and filter the most appropriate resources to be used, based on their educational and pedagogical agenda, while taking into account various limitations (Bernstein, 2015; Cannell, Macintyre, & Hewitt, 2015; Clements & Pawlowski, 2012; Pirkkalainen & Pawlowski, 2013; Proctor & Marks, 2013). Therefore, in spite of the great potential of such repositories in promoting teaching and learning, their use is relatively low among teachers.

Thus, the main goal of the proposed research is to foster a long-lasting pedagogical change among mathematics teachers regarding the use of MAPPs in the teaching sequence (Bielaczyc, 2006) with a resulting enhancement to students' learning. To achieve this goal, we have formulated the following research questions:

1. **Incorporating MAPPs as an integral part of the teaching sequence**
   a) What types of metadata should be provided to the teachers in order to foster their decision making regarding the relevance of MAPPs to the teaching sequence?
   b) How can MAPPs be systematically integrated into the teaching sequence in a structured way, based on teachers' goals?
2. **Data-driven decision-support tools for teachers**
   a) How to visualize information for teachers in order to help them realize students' actual MAPPs usage, and to allow them to provide ad hoc support to students' learning, as necessary?
   b) How can teachers be provided with log-based recommendations in order to help them personalize students' further learning with MAPPs in an informed way?
3. **Students' learning**
   c) What are the learning trajectories of students whose teachers participated in the research, compared to learning trajectories of students of non-research teachers who used the same MAPPs (as measured using data from the MAPP environment)?
   d) What are the learning gains of students whose teachers participated in the research, compared to learning gains of students of non-research teachers (based on external assessment, the Meitzav national exams)?

Scientific background

Teaching Mathematics in a Technological Environment

In recent years, studies have examined mathematics teaching in technological environments that are accessible to teachers and their students. A combination of complementary points of view may be used to address the complexity of the pedagogical work: 1) teachers' knowledge, 2) teachers' work before lessons, and 3) teachers' work during lessons. We relate briefly to each of them.

Teachers' Knowledge: The notion of technological, pedagogical, and content knowledge (TPACK) was suggested by Mishra and Koehler (2006) in an attempt to propose a theory of teacher knowledge for technology integration, based on Shulman's (1986) construct of pedagogical content knowledge (PCK). Shulman (1986) explained that PCK is of specific interest because it identifies the distinctive bodies of knowledge for teaching. Mishra and Koehler (2006) suggested that technological (T) knowledge also needs to overlap knowledge regarding the content to be learned and the pedagogy.

Mishra and Koehler (2006, p. 1029) define TPACK according to the following five characteristics: 1) an understanding of the representation of concepts using technology; 2) pedagogical techniques that use technologies in constructive ways to teach content; 3) knowledge of what makes concepts difficult or easy to learn and how technology can help address some of the problems that students face; 4) knowledge of students' prior knowledge and theories of epistemology; and 5) knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones.

There are three different ways of understanding the TPCK concepts developed in mathematics education (Voogt, Fisser, Pareja Roblin, Tondeur, & van Braak, 2012): T(PCK) as an extension of PCK by integrating technological knowledge; TPCK as a unique and distinct body of knowledge; and TP(A)CK as a knowledge domain that emerges from the integration of technological knowledge, pedagogical knowledge, and content knowledge. While the first two conceptualizations view TPCK as a knowledge domain on its own, TP(A)CK represents an integrative view and emphasizes the relationship between the three knowledge domains and their interconnections. This is the view of TPACK adopted in the current study. One challenge for the suggested research is to help teachers integrate their sound pedagogical and mathematical knowledge with the relatively newer technological knowledge, specifically that of MAPPs.

Teachers' Work Before Lessons: Teachers engage in an ongoing design of teaching sequences for mathematical topics that include a series of classroom activities and homework assignments, including chains of MAPPs, as well as detailed planning of particular lessons. Teachers' ongoing documentation work includes re-sourcing digital and other sources into their everyday practice (e.g., Kynigos, in press; Pepin, Gueudet, & Trouche, (eds.), 2013); note that we use the term "re-source" as proposed by Adler (2000, p.207): "It is possible to think about resource as the verb re-source, to source again or differently." A resource can thus be an artifact (e.g., a textbook), or indeed a discussion with a colleague – anything that may re-source the subject from the "outside", even an artifact that the subject is not aware of (Gueudet, Buteau, Mesa, & Misfeldt, 2014). A second challenge for the proposed research is to help teachers view MAPPs as sources for designing their teaching sequences, and re-sourcing them to the benefit of students' learning.

Teachers' Work During Lessons: The instrumental orchestration framework was proposed by Trouche (2004) and developed by Drijvers and colleagues (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Drijvers, Tacoma, Besamusca, Doorman, & Boon, 2013) to describe teachers' need to support their students during the process of working in technological environments. Whole-class discussions orchestrated by the teacher (Trouche, 2004) can serve as an appropriate forum for talking about and sharing students' personal experiences and insights for the purpose of further enhancing them. Hence, a third challenge for the suggested research is to help teachers to prepare to support students' needs while working with MAPPs and to harness students' (mis)understandings to the benefit of mutual learning.

Decision-Support Tools for Teachers

In recent years, it has been suggested that in the context of computer-based learning, students' logged data might be used to inform teachers of students' learning and to support teachers' decision making (Ben-Naim, Bain, & Marcus, 2009; Dyckhoff, Zielke, Bültmann, Chatti, & Schroeder, 2012; Verber, Duval, Klerkx, Govaerts, & Santos, 2013). Such information may be made available to teachers in visualizations that enable them to explore students' progress easily, to highlight usage patterns that are otherwise hidden (as student-computer interactions are often not visible to teachers), and overall to monitor the classroom effectively. This way, the teacher can respond in time, either by approaching students in need or by addressing problematic topics in the classroom.

Some systems work in real time, providing teachers with pertinent information about the state of the students' learning as it occurs. For example, the tracking tool used by the MIGEN system (2011) visualizes pre-defined "landmarks" to teachers, which occur when the system detects specific actions or repetitive patterns carried out by the student. Other systems allow teachers to specify situations in the students' interactions such as possible student mistakes and tests that can be triggered by the student (Gueraud et al., 2009). Some systems work post-hoc, and generate reports to teachers based on students' complete interaction histories. These systems do not display the students' activities, but rather summarize performance measures such as the number of hints requested and success rates in problems (Heffernan & Heffernan, 2014; Scheuer & Zinn, 2007).

Data mining techniques have been used to analyze students' performance with pedagogical software. The DataShop (Koedinger et al., 2010) system generates student learning curve reports, error reports, and general student performance reports. Gobert, Sao Pedro, Raziuddin, and Baker (2013) trained predictive models to identify students' planning behavior in microworlds, a simulation-based educational software. Their modelling is based on features such as action frequencies and latency between actions. Amershi and Conati (2009) have used data mining techniques to cluster and classify students' interaction behaviors in computer-based learning as either effective or ineffective for learning. Our work differs from these data mining approaches in that it provides temporal and hierarchical visualizations of students' interaction.

There is significant work on computational models for tracing students' knowledge to determine when skills have been learned (Baker, Corbett, & Aleven, 2008). A family of methods use probabilistic methods and machine learning to model students' skill acquisition in intelligent tutoring systems (Pardos, Gowda, Baker, & Heffernan, 2012). The approaches focus on predicting individual student performance on a given set of problems or to select the set of questions that are deemed most suitable for the students' inferred skill level. We focus on a different problem: Which question should be provided next in order to best advance the student's knowledge, given the student's inferred skill level?

Digital Educational Resources in Mathematics Education

The integration of digital educational resources in mathematics classrooms provides a strategic opportunity to improve the quality of education and to enrich classroom experiences (Xu & Recker, 2012; Kuzle, 2012). Furthermore, they have a positive impact on both cognitive and affective (emotional and motivational) characteristics of learners (Kebritchi, Hirumi, & Bai, 2010; Reed, Drijvers, & Kirschner, 2010), as well as on characteristics associated with teacher behavior in the classroom and beyond (Díaz, Nussbaum, & Varela, 2015).

Over the last two decades, a significant development of repositories containing digital learning resources to benefit teachers has taken place (Libbrecht & Goosen, 2016; Cohen, Kalimi, & Nachmias, 2013). These repositories offer a variety of materials and activities in different formats, ranging from texts and short videos to complete lesson plans. Specifically, a large supply of MAPPs covering various topics for a wide age-range of learners is accessible in digital repositories. Therefore, teachers have the opportunity to use these learning materials and resources to enhance their teaching. The literature shows evidence that the use of these MAPPs has a positive impact on learners (Drijvers, Tacoma, Besamusca, Doorman, & Boon, 2013; Hwang & Wu, 2012; Hershkowitz, Tabach, & Dreyfus, 2016; Kapun, Hallun, & Tabach, submitted).

However, despite the growing availability of digital learning resources in general, and in mathematics in particular, along with the many advantages inherent in them, the level of MAPPs' adoption into common teaching practices remains quite low (Joubert, 2013; Recker et al., 2005). A variety of barriers and challenges appears in the literature (Cannell, Macintyre, & Hewitt, 2015; Bernstein, 2014; Pirkkalainen & Pawlowski, 2013). One of the arguments is that many educators are not aware of the growing learning resource repositories or are not convinced of their usefulness (Cannell, Macintyre, & Hewitt, 2015). In addition, there is concern that the nonavailability of clear data concerning the resources will lead to confusion, inconsistency, and time wasted during searches, due to the lack of a unified system of registration, licensing, and cataloging (Nash, 2005); and to difficulties in assessing resource quality and reliability (Clements, Pawlowski, & Manouselis, 2015).

Therefore, assisting teachers in using these digital resources as part of their instructional activities in class is essential (Matsuda, 2008). Teachers need new tools and strategies that will enable them to quickly and effectively search, find, and select relevant learning resources according to their instructional goals and their students' needs, as well as to integrate them into their teaching sequence in class (Carlson & Reidy, 2004; Recker et al., 2007; Webel, Krupa, & McManus, 2015).

Most teacher education programs devote relatively little time to developing expertise in the design of instruction beyond lesson planning (McKenney et al., 2015). However, today's teachers need to plan lessons that incorporate existing classroom activities and instructional digital resources as one continuum. Two exceptional studies tried to address this need. Chien, Chang, Yeh, and Chang (2012) engaged pre-service STEM teachers in a critical reexamination of the affordances of technology for their teaching practices in terms of subject matter selection, motivation empowerment, information presentation, activity design, and pedagogy transition. The project significantly improved the pre-service teachers' technology competency levels. In a similar vein, Trgalová and Jahn (2013) designed a quality questionnaire for the i2geo repository aiming at framing the analysis of available resources by teachers. Their study focused on teachers' changing ability to evaluate online resources and awareness of quality criteria.

For improving teachers' use of digital resources, professional development programs should discuss this very topic, and search tools should be developed based on metadata and usage history (Carlson & Reidy, 2004). The proposed research addresses these two issues.

Research Goals and Implications

Overall, the research goal is twofold. The first research goal is to promote a meaningful, long-lasting pedagogical change among mathematics teachers through the systematic and structured integration of MAPPs as an important resource in their documentation process. This goal will be achieved by supporting the teachers with empirically based practices and data-driven tools to enhance their decision-making regarding the incorporation of MAPPs in class and across the curriculum.

The second research goal is to enhance students' learning by using MAPPs. We aim to improve students' learning trajectories in mathematics. This goal will be achieved by expanding the students' use of MAPPs and by supplying the teachers with recommendations that will address the large variance that is common in the mathematics classroom.

The proposed research has both practical and theoretical implications, which take pedagogical, cognitive, and affective aspects into account. We identify four important implications:

1. Characterizing ways to integrate MAPPs in mathematics education in a well-informed manner, while encouraging mathematical thinking and increasing students' interest in this field.
2. Empowering mathematics teachers as leaders, by supporting them with practices and tools that will assist them to navigate through the wide range of available digital resources, to wisely choose those that best fit their educational agenda, and to effectively use them.
3. Creating a professional development program model for mathematics teachers that specifically addresses the orchestration of available digital resources in the teaching sequence.
4. Applying advanced research methods, based on log file analysis (namely, learning analytics or educational data mining), to extract important insights from students' interactions with the digital environment.

Research Description

Hypotheses

Considering the importance of MAPPs for teaching and learning, and in light of the limited use of these digital tools (as argued above), the main goal of the proposed research is to achieve a sustainable use of MAPPs in the teaching sequence. Consequently, our hypotheses are:

1. Following the design of data-driven decision-supporting tools and participation in a professional development program, mathematics teachers will be able to orchestrate a systemic, coherent student-use of MAPPs in class, across the curriculum.
2. Orchestration of systemic, coherent student-use of MAPPs in class, across the curriculum, will enhance students' learning and improve learning gains.

Research Plan and Methods

The research is to be completed over three years; its design is mostly based on the above-mentioned research questions. The research timeline and milestones are detailed below, according to the research questions, and are summarized in a Gantt chart that follows the description.

Developing a Framework for Choosing MAPPs for Teaching (Research Question 1a, b, Years 1-2)

It is our plan to develop a framework for choosing MAPPs for teaching. In order to better understand which MAPP metadata teachers might find useful in making well-informed decisions about teaching sequence resource use, we will first explore the available metadata, as well as identify crucial missing information. The metadata may include MAPP topic, appropriate grade-levels, learning goals, needed prerequisite knowledge, mathematical representations that are used, types of interaction, expected duration, audiovisual properties, etc. Then, we will take a qualitative approach by interviewing teachers before and after using MAPPs in their classroom (N=6 teachers).

Professional Development Program (Research Question 1b, Years 1-3)

A major component of the research will be the development, application, evaluation, and modification of a professional development (PD) program that will instruct mathematics teachers regarding the use of MAPPs in their teaching. The initial formation of the PD program will be based on case studies of six teachers from two schools, who will be repeatedly interviewed and observed in their classes during the school year. The purpose of the interviews and observations will be to discuss the best ways to integrate MAPPs into the lessons and across the curriculum. In its first instance during Year 2, the PD program will be offered to a large group of teachers (N=30). After evaluating its effectiveness and modifying it, based on the participants' feedback, we will offer an improved program for another group of teachers (N=30), in hope that this second instance during Year 3 will already present a model for similar, future programs.

Developing Data-Driven Decision-Support Tools for Teachers (Research Questions 2a, b, Years 1-3)

We will supply teachers with data-driven analytical and support tools that will facilitate the integration of MAPPs in their classrooms and increase their understanding of students' interactions with the MAPPs. We will provide two types of data-driven support to teachers, based on learning analytics: one that recommends MAPPs for students based on previous interactions, and one that analyzes and visualizes students' interactions with MAPPs. The first support tool will provide recommended MAPPs that are adapted to the learning needs of individual students. We will trace students' interactions and use computational tools to model the skill acquisition of each student over time. Based on this personalized "student model", we will design an algorithm for selecting MAPPs based on students' inferred skill mastery. The algorithm will use a set of possible MAPPs as input for each student, along with that student's individualized student model. It will rank the MAPPs in the set based on the predicted score that is given by the student model. It will select the MAPPs that are intended to advance the student's knowledge of a relevant skill, while matching the student's inferred capabilities, maintaining the "zone of possible achievement" for that student. A key challenge to this method will be building the personalized student models from the log data. We will use two approaches for this purpose: The first approach will be based on tracing students' knowledge. We will extend existing computational tools for knowledge tracing, which have been used on closed, more constrained settings (such as multiple choice questions), to the less constrained and more exploratory MAPP setting. The second approach will be based on combining methods from recommendation systems and information retrieval to infer a difficulty ranking over the set of MAPPs. This method works by aggregating the known difficulty rankings over questions solved by other, similar students. Both of these approaches will be validated empirically using real log data that will be collected as part of our research activities.

The second support tool will consist of visualization tools that will analyze students' interactions with MAPPs, and detect "critical patterns" in their learning trajectories that may require teachers' intervention. The visualizations will be based on the most informative measures that may assist teachers (see theoretical background section) and on empirical evidence that will be gathered during the previously mentioned case studies with a few teachers (N=6). Specifically, the critical patterns we will detect may include students who exhibit sharply rising or falling performance curves over time, exhibit guessing or random play (as inferred by the system), or unusually long or short dwell times between tasks. At the aggregate level, we will provide teachers with real time notification of students' use of MAPPs; including a "heat-map" distribution of students' inferred mathematical skill levels using the knowledge tracing tools from the last section. We will present this heat-map as a histogram that will allow teachers to quickly identify groups of students whose learning trajectories exhibit sharply positive or negative slopes, and enable them to respond accordingly.

Comparing Students' Learning (Research Questions 3a, b, Years 2-3)

Assessment of the effects of MAPPs usage on students' learning will be conducted in two ways. First, evaluation of students' learning trajectories will be made as evident in the MAPPs. That is, students' performance evaluation will be based on their achievement within the MAPPs and on how their competency has progressed. After developing the exact methods for evaluating learning trajectories, we will compare between two groups of students: those whose teachers participated in the research during Year 2 (N=~900 students), and others (approximately the same sample size) who learned with similar MAPPs and non-research teachers. Data collection will be based on the system log files.

Second, we will compare a sub-set of the students' gains by using an external assessment, the Meitzav examination, which is Israel's national standard assessment in mathematics. Mathematics achievements on the Meitzav examination of fifth-grade students whose teachers participate in the research during Year 2 (N=~300 students) will be compared to students of non-research teachers of the same grade-level and of similar demographics. The Meitzav examination data is accessible via Israel's Ministry of Education's Virtual Research Room[6].

[6] The Virtual Research Room provides researchers with access to empirical data about various aspects of Israel's education systems, http://cms.education.gov.il/EducationCMS/Applications/spss/dafault.htm [in Hebrew; accessed July 2016].

Research Question	Milestone	Y1-I	Y1-II	Y1-III	Y2-I	Y2-II	Y2-III	Y3-I	Y3-II	Y3-III
RQ1a	Metadata exploration	■								
	Interviews with teachers (pre, post)	■		■						
RQ1b	Case studies in 2 schools (~6 teachers)	■	■	■						
	PD program design			■						
	PD program for ~10 schools (~30 teachers)				■	■				
	PD program evaluation and modification						■			
	PD program expansion for additional ~30 teachers							■	■	■
RQ2a	Developing measurements to be visualized	■								
	Constructing visualizations at the individual and aggregate levels		■	■						
	Evaluating visualizations in the classroom				■	■				
	Modifying visualizations						■			
RQ2b	Modeling student performance				■					
	Developing probabilistic-based and rule-based sequencing algorithms					■	■			
	Evaluating sequencing algorithms in the classroom							■		
	Teacher interviews and modification of models								■	■
RQ3a	Developing methods to measure learning trajectory				■	■	■			
	Data collection (log-based)						■			
	Data analysis							■	■	
RQ3b	Data collection (MoE's Virtual Research Room)							■		
	Data analysis								■	■
	Report writing			■			■		■	■

Gantt chart for the proposed research, based on research questions, milestones, and trimesters

Research Population

The qualitative parts of the research (RQ 1a, b) during Year 1 will involve approximately six teachers from two elementary schools in Israel that operate a one-to-one computing program in the classroom. During Year 2, the research population will be extended to approximately 10 schools and 30 teachers that will participate in the PD program (RQ 1b); observations will be completed during these teachers' lessons (RQ 2a) and learning trajectories and gains will be collected for these teachers' students (~900 students, RQ 3a). The schools will be selected from two of the Ministry of Education's districts, based on a convenience sampling of schools that operate a one-to-one computing program in the classroom. Additionally, students' learning trajectories and gains data will be collected from a control group of about 10 schools (~900 students, RQ 3a) that operate similar one-to-one computing programs and will not take part in the PD. In general, all the school cohorts will have similar demographics. Finally, the fifth-grade students from the schools who will participate in the PD (~300 students) will take the Meitzav exams during Year 2 of the research; these students' achievements will be compared with national achievements (RQ 3b). Another extension of the PD will take place during Year 3 with a new group of 10 schools (~30 teachers).

Approvals Needed

In order to successfully conduct the research as planned, we will need to obtain the following approvals:

- Approval is needed from Israel's Ministry of Education for collecting data in schools. Data collected will include teachers' interviews, class observations, and log files. Throughout these data collections, students' personal information will either be completely non-available (e.g., during observations) or made anonymous before it reaches the research team (e.g., data from log files).

- Approval is needed from Israel's Ministry of Education for collecting data from the Virtual Research Room.
- Approval is needed from the Institutional Ethics Committee for the research plan. This Committee makes sure that each research proposal it receives is set to the highest degree of ethics in research, mainly concerning participants' privacy.

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