

Mathematics

5 Points – 1st Exam

Instructions

- א. Duration of the exam: Four hours.
- ב. Exam structure and breakdown of points:
This exam has three parts and a total of eight questions.
Part One: Algebra and Probability
Part Two: Planar Geometry and Planar Trigonometry
Part Three: Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions
Answer five questions of your choice – 22 points for each question.
The maximum number of points you can earn is 100.
- ג. Material that may be used during the exam:
(1) A non-graphing calculator.
Do not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language/foreign language–Hebrew dictionary.
- ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or may lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.
If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

5 יחידות לימוד – שאלון ראשון

הוראות

- א. משך הבחינה: ארבע שעות.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון – אלגברה והסתברות
פרק שני – גאומטריה וטריגונומטריה במישור
פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות
יש לענות על חמש שאלות לבחירתכם – 22 נקודות לשאלה.
סך הנקודות שתוכלו לצבור לא יעלה על 100.
- ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי/לועזי–עברי.
- ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer five of the questions 1–8 (each question – 22 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra and Probability

1. On Sunday, two cars – Car 1 and Car 2 – left Town A and traveled the same road to Town B .
 Car 1 left at 8:00 and traveled at v km/h, and Car 2 left at 8:27 and travelled at mv km/h.
 Both cars arrived at Town B at the same time.
- א. Determine which of the following expressions I–III below represents the length of the road between Town A and Town B . Explain your answer.

I $\frac{0.45mv}{m-1}$

II $\frac{1.45mv}{m+1}$

III $\frac{0.55mv}{m-1}$

On Monday at 8:00 Car 1 left Town B and traveled to Town A , and Car 2 left Town A and traveled to Town B .

Both cars traveled on the same road, at the same speed, as they had traveled on Sunday.

45 minutes after both cars set off, the distance between them was 36 km.

The cars met at 9:00 .

- א. (1) Find the value of m .
 (2) Find the length of the road between Town A and Town B .

Immediately after the cars met, Car 1 stopped for one and a half hours and then continued to Town A at the same speed as before.

Car 2 continued traveling without stopping, and as soon as it reached Town B it turned back and traveled to Town A .

- א. Find at what time the cars met the second time.

2. a_n is a decreasing infinite geometric sequence of common ratio q whose terms are all positive.
 א. Consider the following statements I–III. Determine which of the statements is correct, and explain your answer.

I $-1 < q < 0$

II $0 < q < 1$

III $1 < q$

b_n is an infinite sequence whose terms satisfy $b_n = a_n + a_{n+1} + a_{n+2}$ for all natural n .

- א. Prove that Sequence b_n is geometric and that its common ratio is q .

We know that the sum of Sequence b_n is 1.96 times greater than the sum of Sequence a_n .

- א. Find the value of q .

c_n is a sequence whose terms satisfy $c_n = \frac{b_n}{4 \cdot a_n}$ for all natural n .

We know that the sum of the first k terms in the sequence is 34.79.

- א. (1) Prove that the sequence c_n is constant.
 (2) Find the value of k .

We now reverse the sign of all the terms in the even positions in Sequence c_n .

- א. Find the sum of the first k terms in Sequence c_n after the signs were reversed. Explain your answer.

3. Candidates who want to be hired by a certain company must pass three application steps in the following order: Test A, Test B, and a personal interview. Only candidates who pass Test A can take Test B, and only candidates who pass Test B get a personal interview.
 Candidates who pass all three steps are hired.

The probability of a candidate passing Test A is p ($0 < p < 1$).

The probability that a candidate who passed Test A will pass Test B is 2 times greater the probability of a candidate passing Test A.

The probability that a candidate will be hired is 3 times greater than the probability that a candidate will pass both tests but not the interview.

- א. Find the probability that a candidate who passed both tests will pass the interview.

94% of the candidates are not hired.

- א. Find the value of p .
 א. We know that a certain candidate was not hired. What is the probability that this candidate passed Test A?

5 candidates are randomly selected from among all the candidates who were not hired.

- א. What is the probability that at least two of them passed Test A?

Part Two – Planar Geometry and Planar Trigonometry

4. ABC is an equilateral triangle [משולש שווה צלעות]
inscribed in a circle.

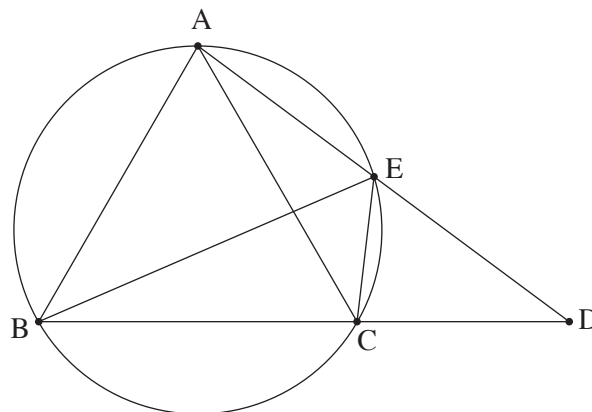
Point D is located on the extension of Side BC ,
as shown in the figure.

Segment AD intersects the circle at Point E .

א. Prove that $\angle AEB = \angle BEC = \angle CED = 60^\circ$.

ב. Prove that $\triangle AEB \sim \triangle CED$.

ג. Prove that $\frac{AE}{CE} = \frac{BC}{CD}$.



We know that the area of Triangle AEB is 2.25 times greater than the area of Triangle CED .

ד. Find by how many times the area of Triangle ABD is greater than the area of Triangle CED .

5. The figure on the right describes an isosceles triangle [משולש שווה שוקיים] ABC ($AB = AC$) .

Point E is located on Base BC such that $BE = 3 \cdot EC$.

Point D is located on Side AC such that ED is
perpendicular to AC .

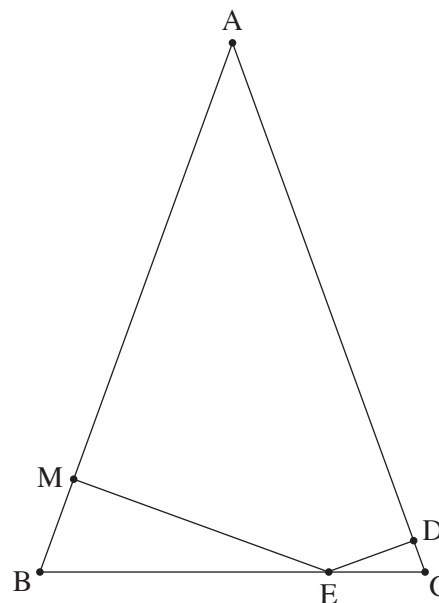
Point M is located on Side AB such that EM is
perpendicular to AB .

Let: $EC = k$, $\angle ABC = \alpha$, $30^\circ < \alpha < 90^\circ$.

א. Express the lengths of the segments DE and ME in
terms of k and α .

We know that the sum of the areas of Triangles CED and
BEM is $1.6k^2$.

ב. Find the value of α .



We know that the length of the radius of the circle circumscribing Triangle AED is 29 .

ג. Find the value of k .

Part Three – Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions

6. Consider the function $f(x) = \frac{6(x-1)^2}{x^2 + a}$.

a is a positive parameter.

א. (1) Find the domain [תחום הגדרה] of function $f(x)$.

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes (if there are any).

ב. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are. Express your answers in terms of a , if necessary.

We know that the y-coordinate of the maximum of function $f(x)$ is 7.5.

ג. Find the value of a .

Answer Items ד–ה for the value of a that you found.

ד. Sketch a graph of function $f(x)$.

$f'(x)$ is the derivative function of function $f(x)$.

Function $g(x)$ satisfies $g(x) = \frac{-f'(x)}{(f(x))^2}$.

ה. (1) Find the domain of function $g(x)$.

(2) Find the positive and negative intervals of function $g(x)$.

(3) Calculate the area bounded by the graph of function $g(x)$ and the x-axis on the interval $-a \leq x \leq 0$.

7. Consider the function $f(x) = (\sin x)^2 \cdot \cos(2x)$, defined on the domain $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$.

א. Prove that function $f(x)$ is even.

ב. Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.

ג. Sketch a graph of function $f(x)$.

The derivative function $f'(x)$ is defined on the domain $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$.

Consider the function $g(x) = \sqrt{a \cdot f'(x)}$.

a is a negative parameter.

ד. (1) Find the domain [תחום הגדרה] of function $g(x)$.

(2) Find the coordinates of the intersection points of the graph of function $g(x)$ with the x-axis.

We know that function $f(x)$ has only one inflection point [נקודת פיתול] in the domain of function $g(x)$.

ה. Sketch a graph of function $g(x)$.

8. The figure on the right describes the graph of function

$$f(x) = (4x - 2)^3 \text{ and the line } y = 3x + b.$$

b is a positive parameter.

Point A is located on the graph of function $f(x)$ such that its x -coordinate is smaller than the x -coordinate of the intersection point of the graph of function $f(x)$ with the given line.

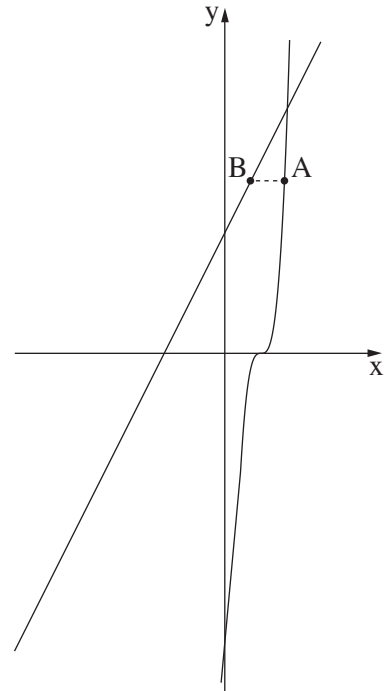
Point B is located on the given line such that Segment AB is parallel to the x -axis.

Let t be the x -coordinate of Point A .

We know that $0 \leq t \leq 0.7$.

א. Express the x -coordinate of Point B in terms of b and t .

- ב. (1) Find the value of t for which the length of Segment AB is minimum.
- (2) Find the value of t for which the length of Segment AB is maximum.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך