

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Two hours and fifty-five minutes.

ב. Exam structure and breakdown of points:

There are two parts in this exam, with a total of five questions.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Exponential and
Logarithmic Functions

Answer three questions of your choice —
 $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות

א. משך הבחינה: שתיים וחמישים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות מעריכיות ולוגריתמיות

יש לענות על שלוש שאלות לבחירתכם —

$$3 \times 33\frac{1}{3} = 100 \text{ נקודות.}$$

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי–לועזי / לועזי–עברי.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider the line $y = x$, and consider the parabola $y^2 = 2px$.

p is a positive parameter.

Point A , located in the first quadrant, is an intersection point of the parabola with the line.

- א. Express the coordinates of Point A in terms of p .

Let ℓ denote the tangent to the parabola at Point A .

Point F is the parabola's focus. Circle I whose center is at Point F is tangent to Line ℓ at Point B .

- ב. Express the equation of Circle I in terms of p .

- ג. Show that Point B is located on the y -axis.

We know that $AB = 4\sqrt{5}$.

- ד. Find the value of p .

- ה. Find the equation of the locus [המקום הגאומטרי] of the centers of the circles that are internally tangent to Circle I , and whose radius is $2\sqrt{5} - 3$.

2. The figure to the right shows Pyramid SABCD whose base ABCD is a square.

Edge AS is a height in the pyramid.

The length of Edge AS is equal to the side length of Square ABCD.

Point E is located on Edge SD such that $SE = \frac{3}{4} \cdot SD$.

Point N satisfies: $\overrightarrow{SN} = k \cdot \overrightarrow{SC}$. k is a parameter.

Let $\overrightarrow{AD} = \underline{u}$, $\overrightarrow{AB} = \underline{v}$, $\overrightarrow{AS} = \underline{w}$.

א. Express the vector $\overrightarrow{EN}$ in terms of $\underline{u}$, $\underline{v}$, $\underline{w}$, and k.

We know that $|\overrightarrow{EN}| = \frac{\sqrt{6}}{4} \cdot |\underline{w}|$.

ב. Find the value of k.

Substitute $k = \frac{1}{2}$, and answer Items ג-ה.

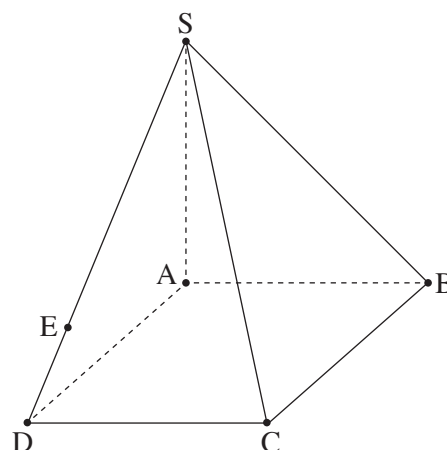
We place the pyramid in a coordinate system such that $A(0, 0, 0)$, $B(0, 12, 0)$.

Vertex D is located on the positive side of the x-axis, and Vertex S is located on the positive side of the z-axis.

ג. (1) Find the coordinates of Points N and E.

(2) Find the equation of Plane ENB.

ה. Is the volume of Pyramid SENB larger than, smaller than, or equal to the volume of Pyramid CENB? Explain your answer.



3. Consider the following three complex numbers:

$$z_1 = r \cdot (\cos \alpha + i \cdot \sin \alpha), \quad z_2 = \frac{1}{r} \cdot (\cos(\alpha + 90^\circ) + i \cdot \sin(\alpha + 90^\circ)), \quad w = z_1 \cdot z_2 + \frac{z_1}{z_2}.$$

$r > 0$, $0^\circ < \alpha < 90^\circ$.

Point A represents the number w in the Gauss plane. We know that the x-coordinate of Point A is: -1.

א. Find the value of α .

We know that Point A is located on the line $y = -1$.

ב. Find the value of r.

Consider the equation $z^4 = 128 \cdot (z_1 \cdot w)^2$.

z is a complex variable.

ג. Find the four solutions of the equation.

Point O is the origin.

Each of the points B and C represents one solution from among the solutions you found in Item ג.

Point B is located in the first quadrant and Point C is located in the second quadrant.

The locus [המקום הגאומטרי] of all the points $z = x + iy$ in the Gauss plane which satisfy $|z| = b$ intersects the positive part of the y-axis at Point K.

b is a positive real number.

We know that the area of Quadrilateral CKBO is $7\sqrt{2}$.

05 ה. Find the value of b.

(Continued on page 4)

Part Two — Growth and Decay, Exponential and Logarithmic Functions

4. Consider the function $f(x) = \frac{1}{e^x - 1} + \frac{5}{e^x - 5} + a$.

a is a parameter.

א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

Express your answers in terms of a , if necessary.

ב. Prove that the derivative function $f'(x)$ is negative on its entire domain.

We know that the line $y = 2$ is an asymptote of function $f(x)$.

ג. Find the two possible values of a .

Substitute $a = 2$ in function $f(x)$, and answer Items ד–ו.

ד. Show that $f(x) = \frac{2e^{2x} - 6e^x}{e^{2x} - 6e^x + 5}$.

ה. (1) Find the coordinates of the intersection point of the graph of function $f(x)$ with the x -axis.

(2) Sketch a graph of function $f(x)$.

ו. Calculate the area bounded by the graph of function $f(x)$, the x -axis, and the line $x = \ln 2$.

5. Consider the function $f(x) = 8 \cdot \ln(x^2 + 7) - 2x$, defined for all x .

א. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.

ב. Find the intervals on which function $f(x)$ is concave upward [קעירות כלפי מעלה] and the intervals on which it is concave downward [קעירות כלפי מטה].

We know that function $f(x)$ intersects the x -axis at only one point.

ג. Sketch a graph of function $f(x)$.

Consider the function $g(x) = \ln(f(x) - 18)$, defined on the domain [תחום ההגדרה] where $f(x) > 18$.

ד. (1) How many asymptotes perpendicular to the x -axis does function $g(x)$ have? Explain your answer.

(2) Find the coordinates of the extremum [נקודת קיצון] of function $g(x)$, and determine what type of extremum it is.

(3) Sketch a graph of function $g(x)$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך