

Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions

הוראות

א. Duration of the exam: Two hours and twenty minutes.

א. משך הבחינה: שתיים ועשרים דקות.

ב. Exam structure and breakdown of points:

ב. מבנה השאלון ומפתח ההערכה:

This exam has two parts, with a total of five questions.

בשאלון זה שני פרקים, ובהם חמש שאלות.

Part One – Sequences, Spatial Trigonometry

פרק ראשון – סדרות, טריגונומטרייה במרחב

Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

פרק שני – גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה

Answer three questions of your choice

יש לענות על שלוש שאלות לבחירתכם

$$- 3 \times 33\frac{1}{3} = 100 \text{ points.}$$

$$- 3 \times 33\frac{1}{3} = 100 \text{ נקודות.}$$

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) Formula sheets (attached).

(2) דפי נוסחאות (מצורפים).

(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

(3) מילון עברי–לועזי / לועזי–עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each question on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

Lack of detail may lower your score or lead to the disqualification of your exam.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.

If you use any draft paper outside the answer booklet, your exam may be disqualified.

כתבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Sequences, Spatial Trigonometry

Sequences

1. The students of Class 12A and the students of Class 12B at a certain high school decided to hold a graduation party.

To cover the cost of the party they collected empty bottles over the course of several weeks, and then handed the bottles in for recycling in exchange for money.

Class 12A collected bottles over the course of 5 weeks.

The number of bottles they collected every week was q times greater than the number of bottles they had collected the previous week.

The number of bottles Class 12A collected in the fourth week was 3.375 times greater than the number of bottles they had collected in the first week.

- א. Find the value of q .

The total number of bottles that Class 12A collected in the first week and the last week was 2,425.

- ב. Find how many bottles Class 12A collected in the first week.

Class 12B collected bottles over the course of 18 weeks. Every week they collected d bottles more than they had collected the previous week.

In the second week Class 12B collected 230 bottles.

Class 12B collected a total of 6,300 bottles.

- ג. Find the value of d .

The students received 0.3 shekels for each bottle they handed in for recycling.

The cost of the graduation party is 3,400 shekels.

- ד. Is the total sum that the students of both classes received from recycling all the bottles enough to cover the cost of the party? Explain your answer.

Spatial Trigonometry

2. The figure to the right shows a right pyramid $SABCD$ whose base $ABCD$ is a square. SO is a height of the pyramid.

We know that the length of SO is 7.2 and that the volume of the pyramid is 38.4 .

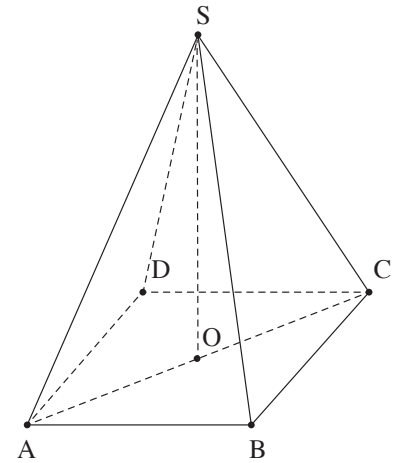
- א. Find the length of an edge of Base $ABCD$.
- ב. Find the size of the angle between a side edge and the pyramid base.

Point F is located on the pyramid height SO .

Segments are drawn from Point F to the vertices A , B , and D , forming a new right pyramid $FABD$.

We know that the size of the angle between a side edge of Pyramid $FABD$ and Base ABD is 60° .

- א. (1) Find the length of Segment FO .
- (2) Find how many times the volume of Pyramid $SABCD$ is greater than the volume of Pyramid $FABD$.



Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

3. Consider the function $f(x) = ax + 5 \cos(2x) - 1$ defined on the domain $0 \leq x \leq \frac{2}{3}\pi$.

a is a parameter.

We know that the slope of the tangent to the graph of function $f(x)$ is -5 at the point where $x = \frac{\pi}{4}$.

- א. Find the value of a .

Substitute $a = 5$ in function $f(x)$ and answer Items ב–ד.

- ב. Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$ and determine what type of extrema they are.
- ג. Sketch a graph of function $f(x)$.

Consider function $g(x)$ whose derivative satisfies $g'(x) = f(x)$.

Function $g(x)$ and the derivative function $g'(x)$, are both defined on the domain $0 \leq x \leq \frac{2}{3}\pi$.

- ד. Read Statements I–II. Determine whether each statement is true or false. Explain your answers.

I Function $g(x)$ has two local extrema [נקודות קיצון פנימיות].

II Function $g(x)$ is increasing on the domain $0 < x < \frac{2}{3}\pi$.

4. Consider the function $f(x) = (2x^2 - ax) \cdot e^x$, defined for all x .

a is a parameter.

We know that function $f(x)$ has an extremum [נקודת קיצון] at the point where $x = 1$.

- א. Find the value of a .

Substitute $a = 3$ in function $f(x)$ and answer Items ב–ד.

- ב. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$ and determine what type of extrema they are.
- ג. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
- ד. Sketch a graph of function $f(x)$.
- ה. Find the positive and negative intervals of function $f(x)$.

Consider the function $g(x) = -f(x) - 2e$, defined for all x .

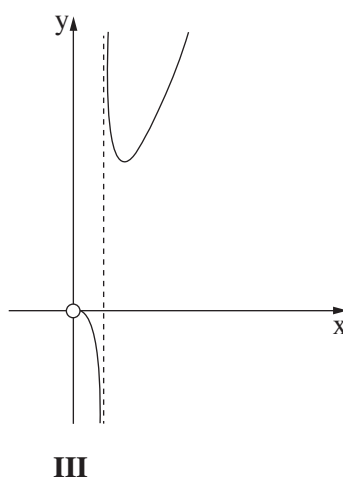
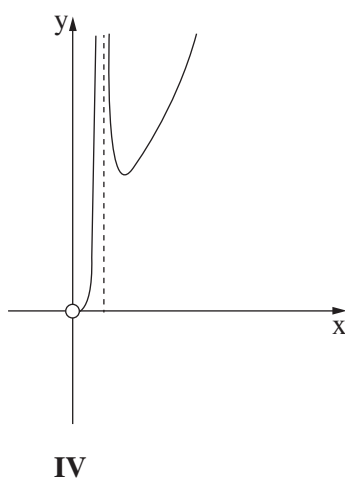
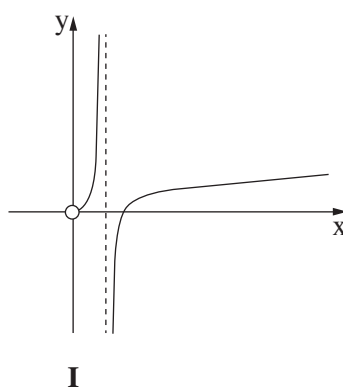
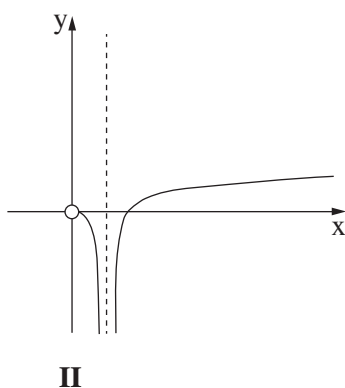
- ו. (1) Find the coordinates of the extrema of function $g(x)$ and determine what type of extrema they are.
- (2) Does the graph of function $g(x)$ intersect the x -axis? Explain your answer.

5. Consider the function $f(x) = \frac{x^2}{-6 + 4\ln(x)}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
 (2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x-axis.
 ב. Does the graph of function $f(x)$ intersect the x-axis? Explain your answer.
 ג. (1) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
 (2) Find the intervals on which function $f(x)$ is increasing and decreasing.

The domain of the derivative function $f'(x)$ is the same as the domain of function $f(x)$.

- ד. (1) Determine which of the graphs I–IV below describes function $f(x)$ and which of them describes the derivative function $f'(x)$. Explain your answers.
 (2) Calculate the area bounded by the graph of the derivative function $f'(x)$, the x-axis, and the line $x = e^3$.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך