

Mathematics

4 Points – 1st Exam

Instructions

- א. Duration of the exam: Four hours.
- ב. Exam structure and breakdown of points:
This exam has three parts with a total of eight questions.
Part One: Algebra, Analytical Geometry and Probability
Part Two: Planar Geometry and Planar Trigonometry
Part Three: Differential and Integral Calculus of
Polynomials, Rational Functions, and Root
Functions
Answer five questions of your choice – each question is
worth 22 points.
The maximum number of points you can earn is 100.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language/
foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page.
If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

4 יחידות לימוד – שאלון ראשון

הוראות

- א. משך הבחינה: ארבע שעות.
ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון – אלגברה, גאומטריה אנליטית והסתברות
פרק שני – גאומטריה וטריגונומטריה במישור
פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים,
של פונקציות רציונליות ושל פונקציות שורש
יש לענות על חמש שאלות לבחירתכם – לכל שאלה 22 נקודות.
סך הנקודות שתוכלו לצבור לא יעלה על 100.
ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי / לועזי–עברי.
ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer five of the questions 1-8 (each question – 22 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra, Analytical Geometry and Probability

1. Hanan's house is located between Uri's house and the grocery store, as shown in the figure below.



The distance between Uri's house and Hanan's house is 3 km.

The distance between Hanan's house and the grocery store is 5 km.

Uri set off from home to the grocery store on his bicycle, cycling at a constant speed.

Hanan set off from home to the grocery store on his bicycle, cycling 3 km/h faster than Uri.

Hanan set off from home 20 minutes after Uri set off.

Uri and Hanan arrived at the grocery store at exactly the same time.

א. Find Uri's cycling speed.

When Uri and Hanan arrived at the grocery store they shopped for some groceries.

Uri spent 10 minutes at the store, and then immediately cycled back home at the same speed he had cycled before.

Hanan spent 12 minutes at the store, and then immediately cycled back home at the same speed he had cycled before.

ב. Find at what distance from the store Uri and Hanan met on their way back.

2. The figure on the right describes a circle. The center of the circle, M , is located in the second quadrant.

The circle passes through the origin, O .

We know that the y -coordinate of Point M is 8,
and that the radius of the circle is 10.

- א. (1) Find the x -coordinate of Point M .
- (2) Find the equation of the circle.

Point B is the intersection point of the circle with the negative side of the x -axis.

- ב. Find the coordinates of Point B .

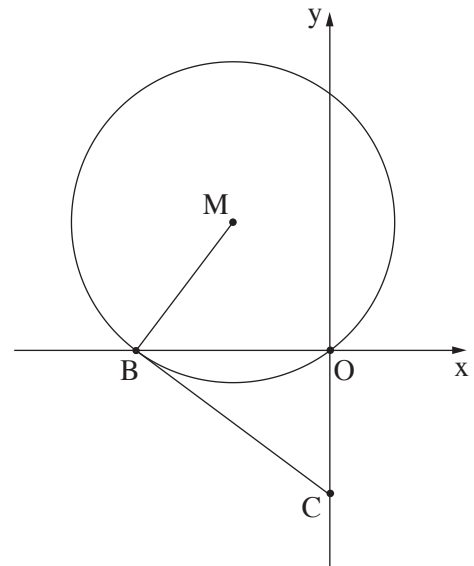
A tangent to the circle is drawn through Point B .

The tangent intersects the y -axis at Point C .

- ג. (1) Find the equation of the tangent.
- (2) Find the length of Segment BC .

A line is drawn through Point M , parallel to Tangent BC and intersecting the y -axis at Point E .

- ד. Calculate the area of Trapezoid $BCEM$.



3. A certain factory manufactures computers.

All the computers manufactured in one particular production cycle underwent two quality inspections to ensure they were operational.

In the first inspection, 80% of the computers passed.

If a computer passes the first inspection, the probability that it will pass the second inspection is 0.9.

If a computer does not pass the first inspection, the probability that it will pass the second inspection is 0.35.

A computer from this production cycle was selected at random.

- א. What is the probability that this computer passed exactly one of the inspections?
- ב. What is the probability that this computer passed at least one of the inspections?

Only a computer that passed both inspections was sent for sale.

A computer from this production cycle was selected at random.

- ג. We know that the selected computer passed at least one of the inspections. What is the probability that this computer was sent for sale?

We know that 250 computers were manufactured in this production cycle.

- ד. Find how many computers were sent for sale in this production cycle.

Two computers manufactured in this production cycle were selected at random one after the other (removal without replacement).

- ה. What is the probability that exactly one computer was sent for sale?

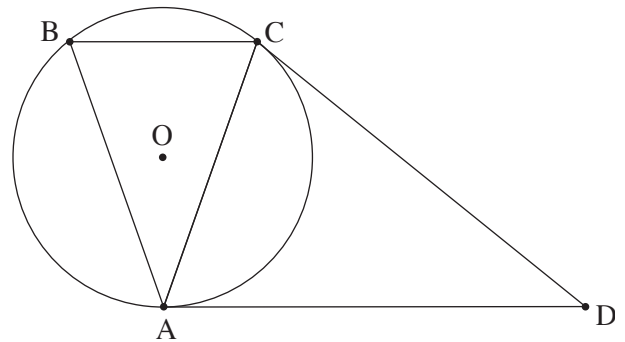
Part Two — Planar Geometry and Planar Trigonometry

4. Triangle ABC is inscribed in a circle of center O .

Segment AD is tangent to the circle at Point A .

Side BC is parallel to Segment AD .

- א. Prove that Triangle ABC is an isosceles triangle [שווה שוקיים].



We know that Segment CD is tangent to the circle at Point C .

- ב. Prove that $\triangle ABC \sim \triangle DAC$.

We know that $BC = 8$, $AD = 18$.

- ג. Find the length of Side AB .

We know that the radius of the circle is $\sqrt{40.5}$.

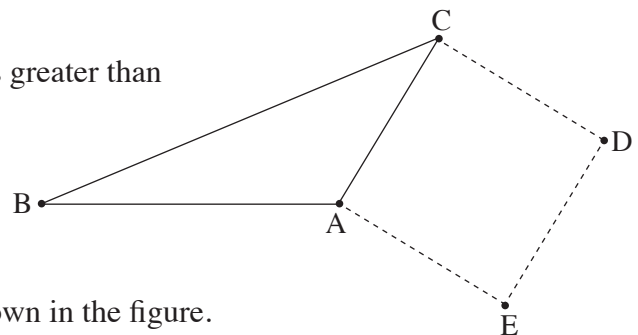
- ד. Find the distance of the Chord AC from the center of the circle.

5. The figure on the right describes Triangle ABC .

We know that the length of Side BC is 2.3 times greater than the length of Side AC ,

$$\angle BAC = 120^\circ .$$

- א. Find the size of Angle ABC .



Square ACDE is constructed on Side AC , as shown in the figure.

We know that the area of Triangle ABC is 25 .

- ב. Find the area of Square ACDE .

- ג. Find the length of Segment BD .

- ד. Find how many times the radius of the circle circumscribing Triangle BCD is greater than the radius of the circle circumscribing Square ACDE .

Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = \frac{4x^2}{x^2 - 9} + a$.

a is a parameter.

א. Find the domain [תחום ההגדרה] of function $f(x)$.

ב. Prove that function $f(x)$ is even.

We know that the graph of function $f(x)$ intersects the x -axis where $x = \sqrt{3}$.

ג. Find the value of a .

Substitute $a = 2$ in function $f(x)$ and answer Items ד-ו.

ד. (1) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.

(2) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.

(3) Find the intervals on which function $f(x)$ is increasing and the intervals on which it is decreasing.

(4) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

ה. Sketch a graph of function $f(x)$.

Consider the function $g(x) = f(x - 7)$.

ו. (1) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.

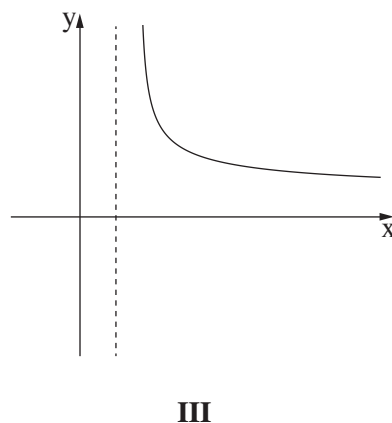
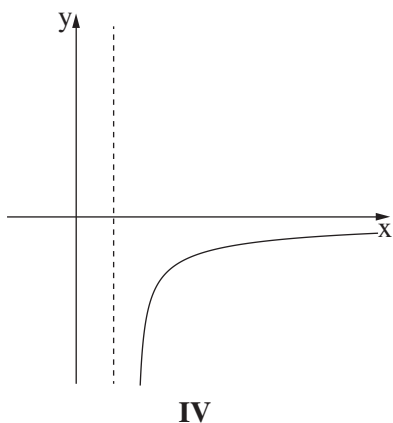
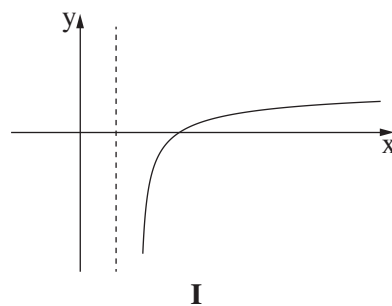
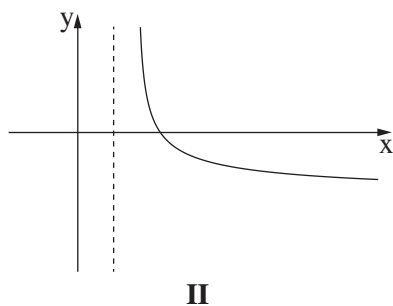
(2) Is function $g(x)$ even? Explain your answer.

7. Consider the function $f(x) = 2x - 4\sqrt{5x - 16}$.

- א. Find the domain [תחום ההגדרה] of function $f(x)$.
- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the x -axis.
- ג. Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- ד. Sketch a graph of function $f(x)$.

The derivative function $f'(x)$ is defined on the domain $x > 3.2$.

- ה. (1) Determine which of the graphs I–IV at the end of the question describes the derivative function $f'(x)$.
- (2) Calculate the area bounded by the graph of the derivative function $f'(x)$, the x -axis, and the lines $x = 4$ and $x = 13$.



8. A landscape architect designed a public park.

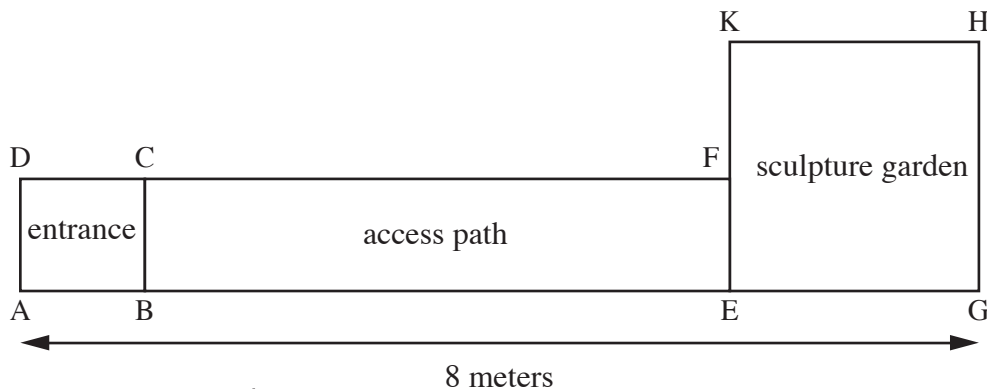
The park consists of three adjoining areas:

an entrance – Rectangle ABCD ;

an access path – Rectangle BEFC ;

a sculpture garden – Square EGHK .

Points A , B , E , and G are located on one line, as shown in the figure.



The length of Segment AG is 8 meters.

The length of Side EG is 2 times the length of Side AB .

Let x denote the length of Side AB .

א. Express the length of Side BE in terms of x .

We know that the area of the entrance is 1 m^2 .

ב. (1) Express the length of Side AD in terms of x .

(2) Express the total area of the park in terms of x .

ג. Find the value of x for which the total area of the park is minimum.

ד. Could the total area of the park be 9 m^2 ? Explain your answer.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך