

Mathematics
4 Points – 2nd Exam

מתמטיקה
4 יחידות לימוד – שאלון שני

Instructions

הוראות

א. Duration of exam: Two hours and twenty minutes.

א. משך הבחינה: שתיים ועשרים דקות.

ב. Exam structure and breakdown of points:

ב. מבנה השאלון ומפתח ההערכה:

This exam has two parts, with a total of five questions.

בשאלון זה שני פרקים, ובהם חמש שאלות.

Part One – Sequences, Spatial Geometry, and Growth and Decay

פרק ראשון – סדרות, גאומטריה במרחב וגדילה ודעיכה

Part Two – Differential and Integral Calculus of Exponential and Logarithmic Functions

פרק שני – חשבון דיפרנציאלי ואינטגרלי של פונקציות מעריכיות ולוגריתמיות

Answer three questions of your choice

יש לענות על שלוש שאלות לבחירתכם

$$- 3 \times 33\frac{1}{3} = 100 \text{ points.}$$

$$- 3 \times 33\frac{1}{3} = 100 \text{ נקודות.}$$

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

(2) Formula sheets (attached).

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי / לועזי–עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each question on a new page.

(2) יש להתחיל כל שאלה בעמוד חדש.

Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

Lack of detail may lower your score or lead to the disqualification of your exam.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.

If you use any draft paper outside the answer booklet, your exam may be disqualified.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5 of your choice (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Sequences, Spatial Geometry, and Growth and Decay

1. The students of Class 12A and the students of Class 12B at a certain high school decided to hold a graduation party.

To cover the cost of the party they collected empty bottles over the course of several weeks, and then handed the bottles in for recycling in exchange for money.

Class 12A collected bottles over the course of 5 weeks.

The number of bottles they collected every week was q times greater than the number of bottles they had collected the previous week.

The number of bottles Class 12A collected in the fourth week was 3.375 times greater than the number of bottles they had collected in the first week.

א. Find the value of q .

The total number of bottles that Class 12A collected in the first week and the last week was 2,425.

ב. Find how many bottles Class 12A collected in the first week.

Class 12B collected bottles over the course of 18 weeks. Every week they collected d bottles more than they had collected the previous week.

In the second week Class 12B collected 230 bottles.

Class 12B collected a total of 6,300 bottles.

ג. Find the value of d .

The students received 0.3 shekels for each bottle they handed in for recycling.

The cost of the graduation party is 3,400 shekels.

- ד. Is the total sum that the students of both classes received from recycling bottles enough to cover the cost of the party? Explain your answer.

2. The figure to the right shows Cuboid $ABCD A' B' C' D'$ whose base $ABCD$ is a square.

Point F is the midpoint of Edge BC .

Point E is located on Diagonal $A' C'$ such that $\overrightarrow{A'E} = \frac{3}{4} \overrightarrow{A'C'}$.

Let $\overrightarrow{AB} = \underline{u}$, $\overrightarrow{BC} = \underline{v}$, $\overrightarrow{AA'} = \underline{w}$.

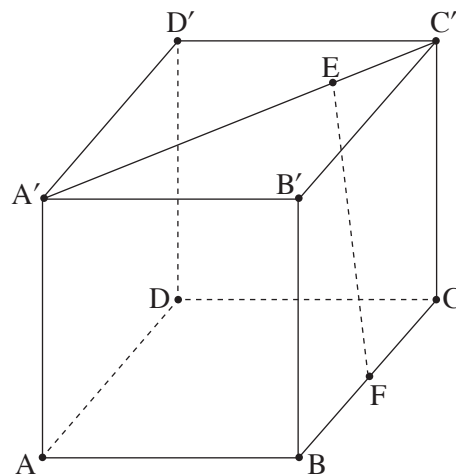
- א. Express the vectors $\overrightarrow{A'E}$ and $\overrightarrow{FE}$ in terms of $\underline{u}$, $\underline{v}$ and $\underline{w}$:

We know that the length of the base edge is 4.

- א. Calculate the length of the vector $\overrightarrow{A'E}$.

We know that $|\overrightarrow{A'E}| = |\overrightarrow{FE}|$.

- א. Show that the cuboid is a cube.
 7. (1) Show that $\overrightarrow{FE}$ is perpendicular to $\overrightarrow{A'C'}$.
 (2) Calculate the area of Triangle $A'FC'$.



3. Penguins inhabit Region A. Since the beginning of 2000 the number of penguins in this region has decreased every year by a fixed percentage.

- א. Is the difference between the number of penguins at the beginning of 2000 and the number at the beginning of 2001 the same as the difference between the number of penguins at the beginning of 2001 and the number at the beginning of 2002? Explain your answer.

At the beginning of 2000 there were 253,000 penguins in Region A. The number of penguins in this region decreases by 5% every year.

Penguins also inhabit Region B. The number of penguins in this region decreases every year by a fixed percentage.

At the beginning of 2000 there were 313,000 penguins in Region B.

At the beginning of 2010 the number of penguins in Region A was the same as the number of penguins in Region B.

- א. Find by what percentage the number of penguins in Region B decreases every year.
 א. Find how long from the beginning of 2000 the number of penguins in Region B was 98,000.

Since the beginning of 2018, the number of penguins in Region A has increased every year by 2%, as a result of the rehabilitation of this region.

7. Find how long from the beginning of 2018 the number of penguins in Region A increased by 17% relative to their number at the beginning of 2018.

Part Two – Differential and Integral Calculus of Exponential and Logarithmic Functions

4. Consider function $f(x) = (3x^2 - a) \cdot e^x$, defined for all x .

a is a parameter.

We know that function $f(x)$ has an extremum [נקודת קיצון] where $x = 1$.

א. Find the value of a .

Substitute $a = 9$ in function $f(x)$, and answer Items ב-ה.

ב. Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

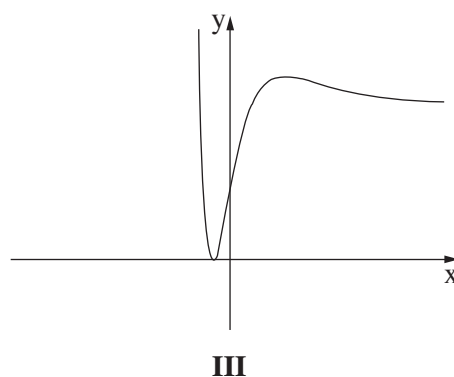
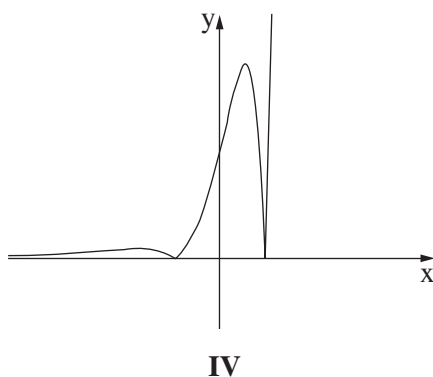
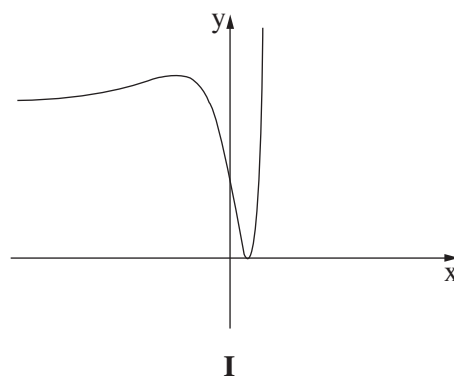
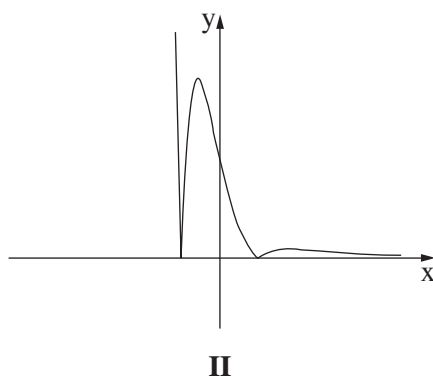
ג. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.

ד. Sketch a graph of function $f(x)$.

Consider the function $g(x) = |f(x)|$, defined for all x .

ה. (1) Determine which of the graphs I–IV below describes function $g(x)$. Explain your answer.

(2) Find the coordinates of the extrema of function $g(x)$, and determine what type of extrema they are.

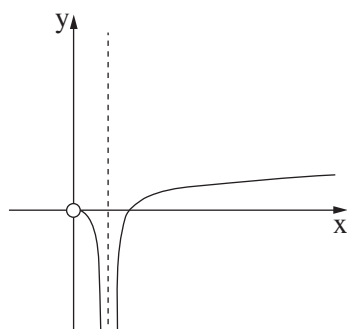


5. Consider the function $f(x) = \frac{x^2}{-6 + 4\ln(x)}$.

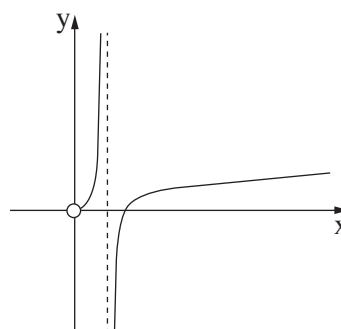
- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x-axis.
- ב. Does the graph of function $f(x)$ intersect the x-axis? Explain your answer.
- ג. (1) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
- (2) Find the intervals on which function $f(x)$ is increasing and decreasing.

The domain of the derivative function $f'(x)$ is the same as the domain of function $f(x)$.

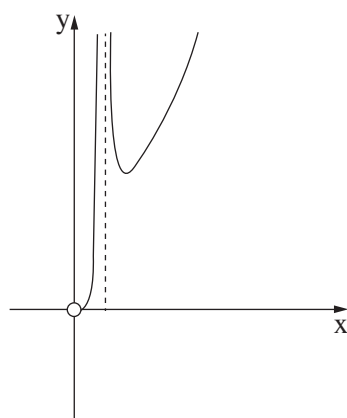
- ד. (1) Determine which of the graphs I–IV below describes function $f(x)$ and which of them describes the derivative function $f'(x)$. Explain your answers.
- (2) Calculate the area bounded by the graph of the derivative function $f'(x)$, the x-axis, and the line $x = e^3$.



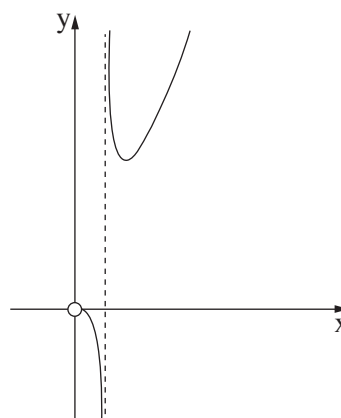
II



I



IV



III

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך