

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד – שאלון ראשון

Instructions

הוראות

א. Duration of the exam: Four hours.

א. משך הבחינה: ארבע שעות.

ב. Exam structure and breakdown of points:

ב. מבנה השאלון ומפתח ההערכה:

This exam consists of three parts, with a total of eight questions.

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

Part One – Statistics and Probability

פרק ראשון – סטטיסטיקה והסתברות

Part Two – Geometry

פרק שני – גאומטריה

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

Answer five questions of your choice – each question is worth 22 points.

יש לענות על חמש שאלות לבחירתכם – כל שאלה 22 נקודות.

The maximum number of points you can earn is 100.

סך הנקודות שתוכלו לצבור לא יעלה על 100.

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות לתכנות.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי / לועזי-עברי.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

(2) יש להתחיל כל שאלה בעמוד חדש.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer five of the questions 1–8 (each question – 22 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Statistics and Probability

1. Researchers conducted a national health study in a certain country.

The study examined the number of steps walked per day by each participant.

They found that the number of steps per day was normally distributed, with a standard deviation of 800 .

They found that 37.5% of study participants walked less than 6,944 steps per day.

- א. Find the study participants' average number of steps per day.

The Ministry of Health recommends walking more than 8,400 steps per day.

- ב. Find the percentage of study participants who follow this recommendation.

The researchers found that 14,722 of study participants walk more than 7,200 steps per day but less than 8,400 steps per day.

- ג. According to the normal distribution table, how many people in total participated in the study?

The researchers also examined the number of hours that each study participant sleeps per day.

They found that the number of hours of sleep per day is also normally distributed.

The study participants' average number of hours of sleep per day is 7 hours.

Sarit participated in the study. Sarit sleeps 9.28 hours per day, and her number of steps per day is 8,416 .

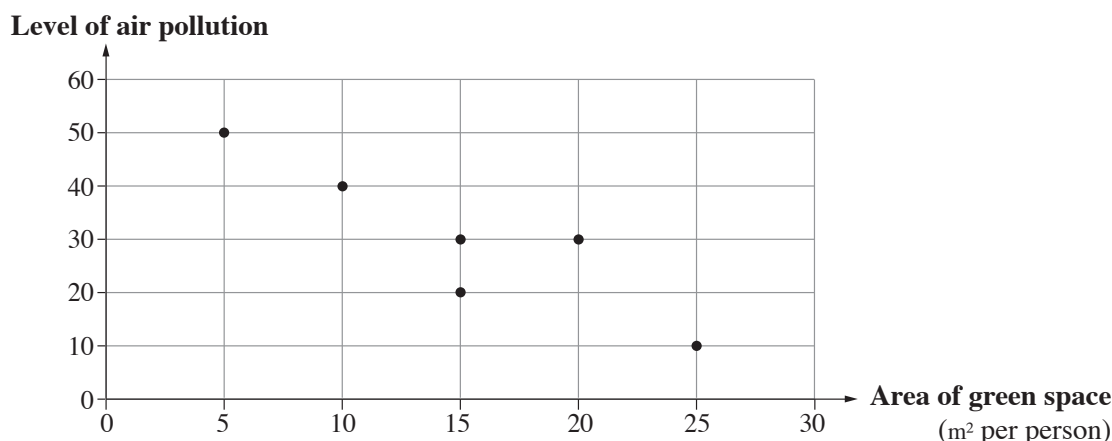
The standard score of Sarit's number of steps per day is identical to the standard score of her number of hours of sleep per day.

- ד. Find the standard deviation of number of hours of sleep per day of the participants in the study.

2. An environmental researcher investigated the relationship between the area of green space (gardens and parks) per city resident (x variable) and the level of air pollution in the city (y variable) in 6 different cities.

The researcher determined the level of air pollution in each city according to the Air Quality Index AQI (a numerical measure of the amount of pollutants in the air).

The scatter diagram below presents the data.



Let r be the correlation coefficient between x and y .

- א. Below are four statements, I–IV. Determine which statement is correct. Explain your answer.

I. $0 < r < 1$

II. $r = -1$

III. $-1 < r < 0$

IV. We cannot determine the range of values for r .

- ב. (1) Calculate the average area of green space per person in the 6 cities.
 (2) Calculate the average level of air pollution in the 6 cities.

We know that the regression line equation for predicting y according to x is $y = -1.8x + b$.

- ג. (1) Find the value of b .
 (2) Based on the regression line, what level of air pollution is predicted in a city in which the area of green space is 18 m^2 per person?
 ד. Calculate the standard deviation of the area of green space in the 6 cities.

The researcher found that the standard deviation of the level of air pollution was $S_y = \sqrt{\frac{500}{3}}$.

- ה. Use the regression line equation for predicting y according to x to find the correlation coefficient r .

The researcher discovered a defect in the measuring instrument and found that, in fact, the actual level of air pollution in each of the 6 cities was higher by 4 than the previously measured level of air pollution. He corrected the data accordingly.

- ו. (1) After this correction, did the slope of the regression line for predicting y according to x increase, decrease, or remain the same? Explain your answer.
 (2) What is the regression line equation for predicting y according to x after correction of the level of air pollution data?

3. A certain factory manufactures computers.

All the computers manufactured in one particular production cycle underwent two quality inspections to ensure they were operational.

In the first inspection, 80% of the computers passed.

If a computer passes the first inspection, the probability that it will pass the second inspection is 0.9 .

If a computer does not pass the first inspection, the probability that it will pass the second inspection is 0.35 .

A computer from this production cycle was selected at random.

- א. What is the probability that this computer passed exactly one of the inspections?
- ב. What is the probability that this computer passed at least one of the inspections?

Only a computer that passed both inspections was sent for sale.

A computer from this production cycle was selected at random.

- ג. We know that the selected computer passed at least one of the inspections. What is the probability that this computer was sent for sale?

We know that 250 computers were manufactured in this production cycle.

- ד. Find how many computers were sent for sale in this production cycle.

Two computers manufactured in this production cycle were selected at random one after the other (removal without replacement).

- ה. What is the probability that exactly one computer was sent for sale?

Part Two – Geometry

4. The figure on the right shows a circle. The center of the circle M is located on the x -axis.

Point A is located on the circle.

The extension of Segment MA intersects the y -axis at Point B .

We know that $A(-16, 6)$, $B(0, 18)$.

א. (1) Find the equation of Line AB .

(2) Find the equation of the circle.

A tangent to the circle was drawn through Point A , intersecting the y -axis at Point D .

ב. Find the coordinates of Point D .

Point C is located on the circle such that AC is a diameter of the circle.

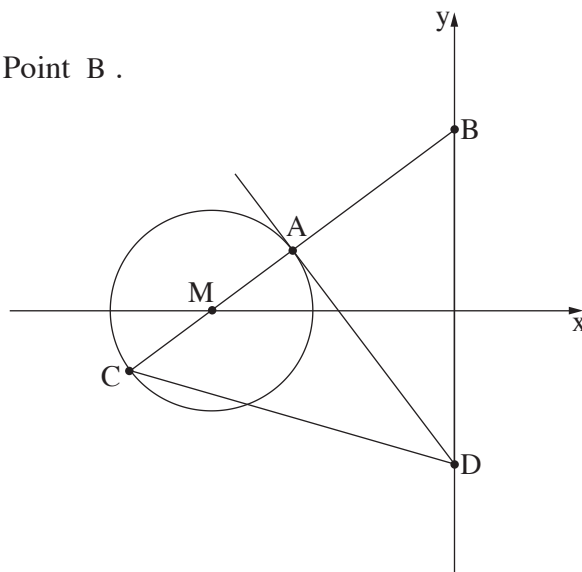
ג. Show that $BD = CD$.

ד. Calculate the size of Angle ABD .

Point K is located on Segment CD .

We know that the area of Triangle MCK is 90.

ה. Find the length of Segment CK .



5. The figure to the right shows a right trapezoid $ABCD$ ($AD \parallel BC$, $\angle ABC = 90^\circ$).

Vertex B is located on the negative side of the x -axis.

We know that $A(0, 4)$, and the length of Side AB is 5.

א. Find the coordinates of Vertex B .

ב. Find the equation of Side BC .

We know that the equation of Diagonal AC is $y = -\frac{1}{3}x + 4$.

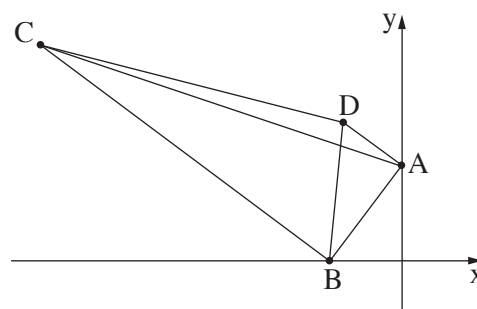
ג. Find the length of Side BC .

Point M is the intersection point of the trapezoid's diagonals.

ד. Prove that $\triangle DMA \sim \triangle BMC$.

We know that the length of Segment AD is 3.

ה. How many times greater is the area of Triangle BMC than the area of Triangle DMA ?



Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = \frac{4x^2}{x^2 - 9} + a$.

a is a parameter.

א. Find the domain [תחום ההגדרה] of function $f(x)$.

ב. Prove that function $f(x)$ is even.

We know that the graph of function $f(x)$ intersects the x -axis where $x = \sqrt{3}$.

ג. Find the value of a .

Substitute $a = 2$ in function $f(x)$ and answer Items 7–10.

7. (1) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.

(2) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.

(3) Find the intervals on which function $f(x)$ is increasing and the intervals on which it is decreasing.

(4) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

ה. Sketch a graph of function $f(x)$.

Consider the function $g(x) = f(x - 7)$.

8. (1) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.

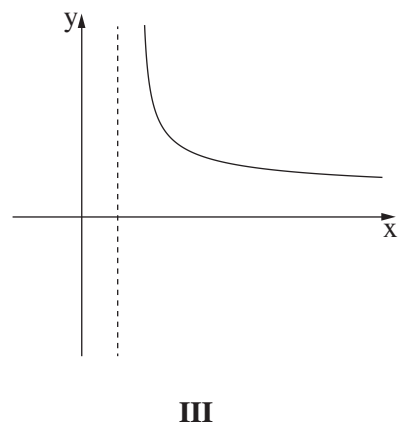
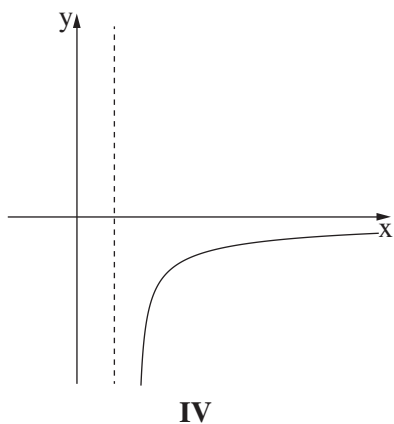
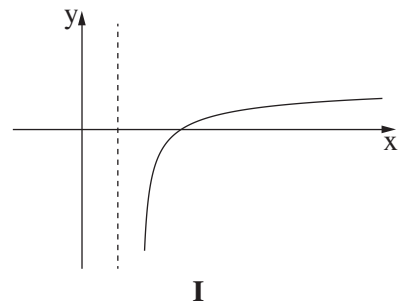
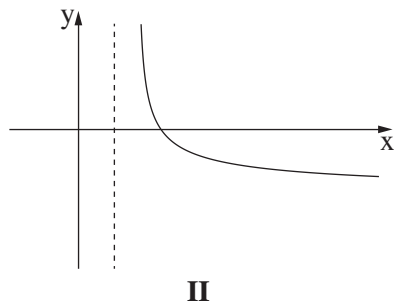
(2) Is function $g(x)$ even? Explain your answer.

7. Consider the function $f(x) = 2x - 4\sqrt{5x - 16}$.

- א. Find the domain [תחום ההגדרה] of function $f(x)$.
- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the x-axis.
- ג. Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- ד. Sketch a graph of function $f(x)$.

The derivative function $f'(x)$ is defined on the domain $x > 3.2$.

- ה. (1) Determine which of the graphs I–IV at the end of the question describes the derivative function $f'(x)$.
- (2) Calculate the area bounded by the graph of the derivative function $f'(x)$, the x-axis, and the lines $x = 4$ and $x = 13$.



8. A landscape architect designed a public park.

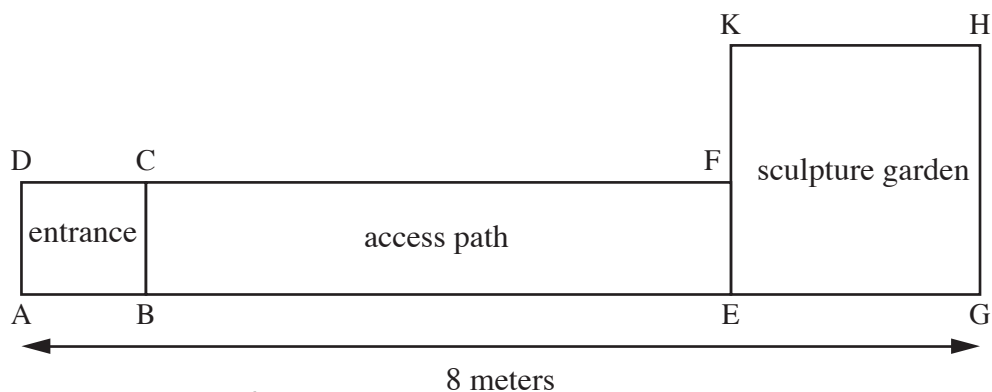
The park consists of three adjoining areas:

an entrance – Rectangle ABCD ;

an access path – Rectangle BEFC ;

a sculpture garden – Square EGHK .

Points A , B , E , and G are located on one line, as shown in the figure.



The length of Segment AG is 8 meters.

The length of Side EG is 2 times the length of Side AB .

Let x denote the length of Side AB .

א. Express the length of Side BE in terms of x .

We know that the area of the entrance is 1 m^2 .

ב. (1) Express the length of Side AD in terms of x .

(2) Express the total area of the park in terms of x .

ג. Find the value of x for which the total area of the park is minimum.

ד. Could the total area of the park be 9 m^2 ? Explain your answer.