

Mathematics

3 Points – 3rd Exam

Instructions

א. Duration of the exam: Two hours and twenty minutes

ב. Exam structure and breakdown of points:

This exam has six questions on the following topics:

Algebra, Differential and Integral Calculus.

Answer four questions of your choice –

each question is worth 29 points.

The maximum number of points you can earn is 100.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

Explain all your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to disqualification of your exam.

Write in the answer booklet only. Write "טייטה" at the top of every page you use for draft work.

If you use pages outside the answer booklet for draft work, your exam may be disqualified.

מתמטיקה

3 יחידות לימוד – שאלון שלישי

הוראות

א. משך הבחינה: שתיים ועשרים דקות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים –

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

יש לענות על ארבע שאלות לבחירתכם –

לכל שאלה 29 נקודות.

סך הנקודות שתוכלו לצבור לא יעלה על 100 נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות

התכנות במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות

במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש.

יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים

מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט

ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.

כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer four of the questions 1–6 (each question – 29 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. Store A sells packets of crayons and sketchbooks.

The price of one packet of crayons is 2 times higher than the price of one sketchbook.

Shahar bought one packet of crayons and 3 sketchbooks in Store A. He paid a total of 160 shekels for this purchase.

- א. Find the price of one sketchbook in Store A.

Store B also sells packets of crayons and sketchbooks.

The price of one packet of crayons in Store B is the same as the price in Store A, but the price of one sketchbook in Store B is 15% lower than the price in Store A.

Noa bought packets of crayons and sketchbooks in Store B.

The number of sketchbooks that Noa bought was greater by 8 than the number of packets of crayons she bought.

Noa paid a total of 673.6 shekels for this purchase.

- ב. (1) Find the price of one sketchbook in Store B.
(2) Find the number of packets of crayons that Noa bought.

Yasmin, the art teacher, wants to buy 48 sketchbooks for her students.

Store A sells packs of 12 sketchbooks. The price of one pack is 312 shekels.

Store B does not sell packs of sketchbooks, but rather sells each sketchbook individually.

The price of each sketchbook in Store B is the price you found in Sub-Item ב(1).

- ג. Determine in which of the stores, Store A or Store B, Yasmin will pay less for the sketchbooks she wants to buy. Explain your answer.

2. The figure to the right shows Quadrilateral ABCD .

Vertex A is located on the y-axis, and Vertex B is located on the x-axis.

Side BC is perpendicular to Side AB .

The equation of Side AB is $y = -\frac{1}{3}x + 2$.

א. Find the coordinates of Vertices A and B .

ב. Find the equation of Side BC .

Side CD is parallel to the x-axis.

We know that D (3 , 6) .

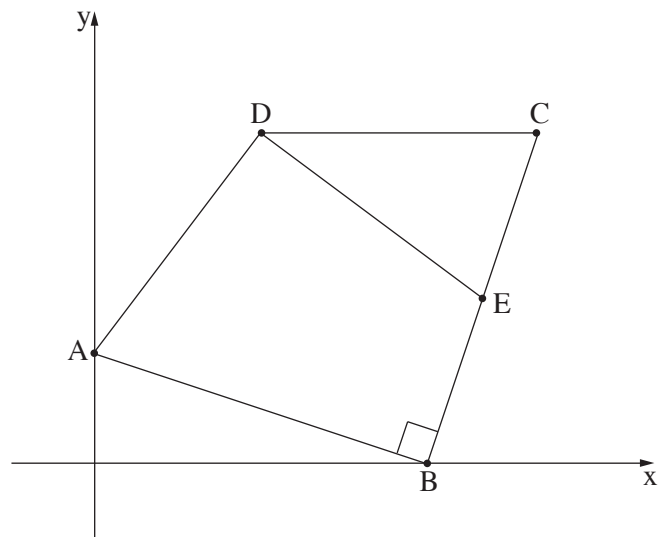
ג. Find the coordinates of Vertex C .

Point E is the midpoint of Side BC .

ד. (1) Find the coordinates of Point E .

(2) Show that Triangle EDC is an isosceles triangle [שווה שוקיים].

ה. Calculate the perimeter of Triangle EDC .



3. The figure to the right shows a circle whose center is Point A .

We know that the equation of the circle is

$$(x - 6)^2 + (y - 3)^2 = 100 .$$

Point B is located on the circle, in the first quadrant.

We know that the x-coordinate of Point B is 12 .

א. What are the coordinates of A , the center of the circle?

ב. Find the coordinates of Point B .

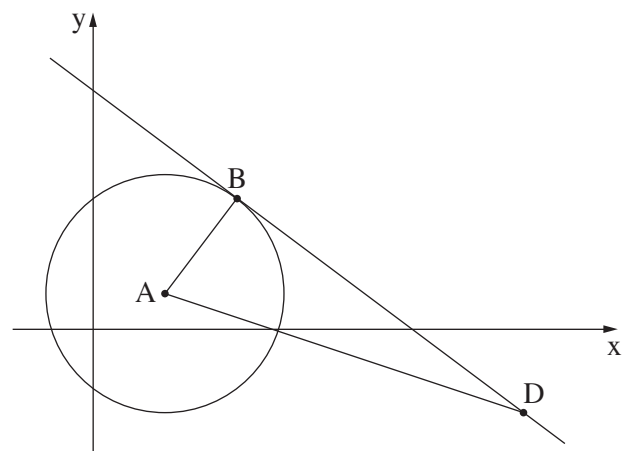
A tangent to the circle was drawn through Point B .

ג. (1) Find the slope of AB .

(2) Find the equation of the tangent .

Point D is located on the tangent, and its y-coordinate is -7 .

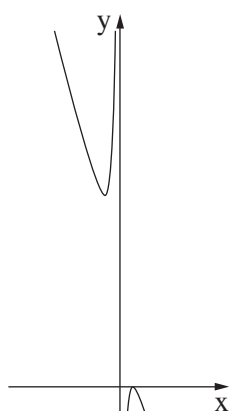
ד. Calculate the area of Triangle ABD .



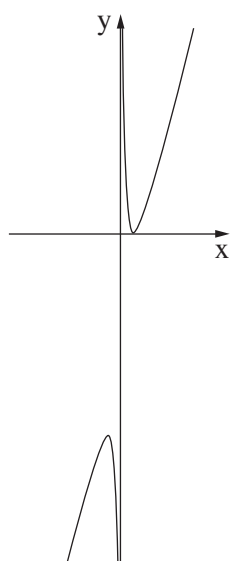
Differential and Integral Calculus

4. Consider function $f(x) = \frac{49}{x} + 4x$.

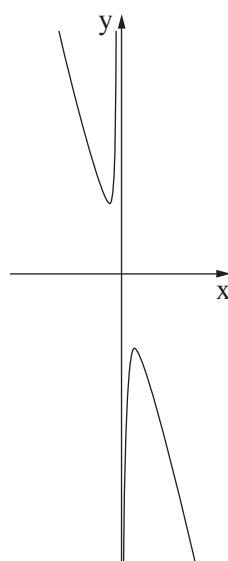
- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x-axis.
- ב. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$ and determine what type of extrema they are.
- ג. Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.
- ד. Determine which of the graphs I–IV below describes function $f(x)$. Explain your answer.
- ה. Choose an example of a point on the graph of function $f(x)$ at which the tangent has a negative slope, and give the coordinates of this point.



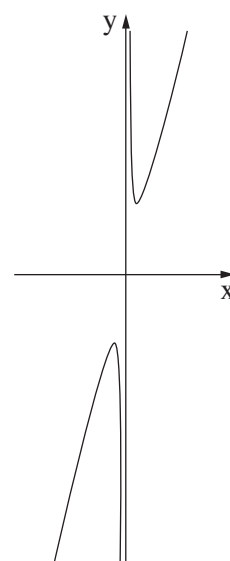
IV



III



II



I

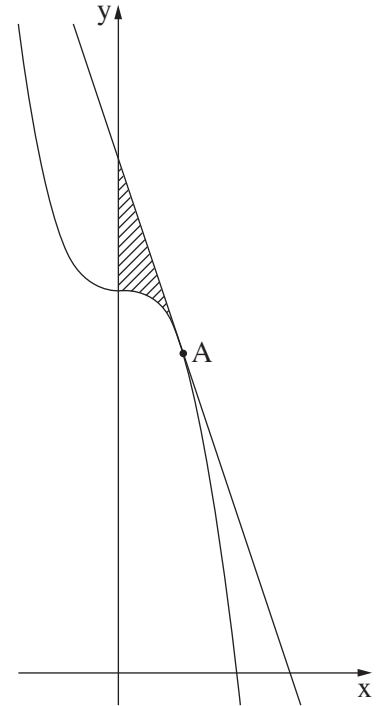
5. The figure on the right describes the graph of function $f(x) = -x^3 + 6$.

Point A is located on the graph of function $f(x)$.

We know that the x-coordinate of Point A is 1.

A tangent to the graph of function $f(x)$ was drawn through Point A.

- א. (1) Find the slope of the tangent.
- (2) Find the equation of the tangent.
- ב. Find the area of the shaded region in the figure:
the region bounded by the graph of function $f(x)$, the tangent,
and the y-axis.



6. A factory produced a window in the shape of Rectangle ABCD.

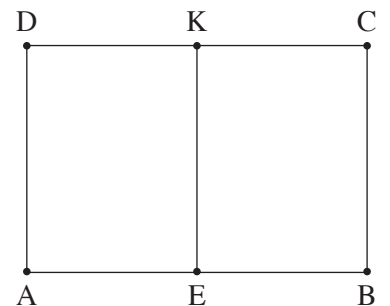
The window consists of four aluminum strips: AB, BC, CD, and AD, and an additional strip of aluminum EK, as shown in the figure.

The length of the aluminum strip EK is the same as the length of the aluminum strip AD.

We know that the combined length of all five aluminum strips is 12 meters in total.

Let x denote the length of the aluminum strip AD.

- א. Express the length of aluminum strip AB in terms of x .
- ב. (1) Express the area of the window ABCD in terms of x .
- (2) Find the value of x for which the area of the window ABCD is maximum.
- ג. Calculate the maximum area of the window ABCD.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך