

Mathematics 5 Points – 1st Exam

מתמטיקה 5 יחידות לימוד – שאלון ראשון

Instructions

הוראות

א. Duration of the exam: Three and a half hours.

א. משך הבחינה: שלוש שעות וחצי.

ב. Exam structure and breakdown of points:

This exam has three parts and a total of eight questions.

Part One: Algebra and Probability

Part Two: Planar Geometry and Planar Trigonometry

Part Three: Differential and Integral Calculus of
Polynomials, Root Functions, Rational
Functions, and Trigonometric Functions

Answer five questions, at least one question from each
part – $5 \times 20 = 100$ points.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

פרק ראשון: אלגברה והסתברות

פרק שני: גאומטריה וטריגונומטריה במישור

פרק שלישי: חשבון דיפרנציאלי ואינטגרלי של פולינומים,
של פונקציות שורש, של פונקציות רציונליות
ושל פונקציות טריגונומטריות

יש לענות על חמש שאלות, לפחות על שאלה אחת מכל פרק
– $20 \times 5 = 100$ נקודות.

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator.

(1) מחשבון לא גרפי.

Do not use the programming options of a programmable
calculator.

אין להשתמש באפשרויות התכנות במחשבון שיש בו
אפשרות תכנות.

Use of a graphing calculator or programming options
may lead to the disqualification of your exam.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון
עלול לגרום לפסילת הבחינה.

(2) Formula sheets (attached).

(2) דפי נוסחאות (מצורפים).

(3) A Hebrew–foreign language/foreign language–Hebrew
dictionary.

(3) מילון עברי–לועזי/לועזי–עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of
the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each question on a new page.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת
את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת
מחשבון.

Write the steps of the solution in your answer booklet,
even when the calculations are done with a calculator.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה
ברורה ומסודרת.

Explain all your work, including calculations, in detail
and in a clear and organized fashion.

חסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Lack of detail may lower your score or may lead to the
disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש

If you write draft material on paper outside the answer booklet, your exam
may be disqualified.

טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

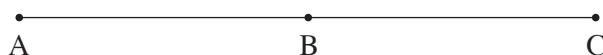
Questions

Answer five of the questions 1–8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Algebra and Probability

1. Towns A, B, and C are located on a line. Town B is located at the midpoint of Segment AC (see figure).



A pedestrian set off from Town B at 9:00 and walked at a constant speed to Town A.

A cyclist set off on his bicycle from Town B at 9:00 and cycled at a constant speed to Town C.

When the cyclist arrived at Town C, he immediately turned back to Town B, cycling at the same speed as before.

When the cyclist arrived back at Town B, the pedestrian had walked $\frac{2}{3}$ of the distance from Town B to Town A.

⌘. Find by what factor the cyclist's speed was faster than the pedestrian's speed.

When the cyclist arrived back at Town B, he rested for one hour, and then set off for Town A, cycling at the same speed as before.

When the pedestrian arrived at Town A, he immediately turned back to Town B, walking at the same speed as before.

6 minutes after the cyclist left Town B to go to Town A, he met the pedestrian who was on his way back from Town A.

Let v be the pedestrian's walking speed.

⌘. Show that the distance between Town A and Town C is $2.1v$.

The next day, the cyclist and the pedestrian set off from Town C to Town A at the same time.

The cyclist cycled at a speed that was 1 km/h slower than his speed the day before, and the pedestrian walked at a speed that was 2 km/h faster than his speed the day before.

The cyclist arrived at Town A 45 minutes before the pedestrian.

⌘. Find the distance from Town A to Town C.

2. a_n is an increasing infinite geometric sequence of common ratio q .
 b_n is an increasing infinite geometric sequence of common ratio $2q$.
 c_n is an infinite sequence whose terms satisfy $c_n = a_n \cdot b_n$ for all natural n .
א. Prove that c_n is a geometric sequence, and express its common ratio in terms of q .

We know that $c_2 = \frac{1}{8} \cdot (a_1)^2$, $a_1 = b_1$.

- א. Find the value of q .
א. (1) Is the value of a_1 positive or negative? Explain your answer.
(2) Is Sequence c_n increasing or decreasing? Explain your answer.

Let S_1 be the sum of Sequence a_n , let S_2 be the sum of Sequence b_n , and let S_3 be the sum of Sequence c_n .

We know that $S_1 + S_2 + S_3 = 434$.

- א. Find the value of a_1 .

3. Bag A and Bag B contain two types of balls: soft balls and hard balls.

Bag A contains 10 balls, of which 4 balls are soft and the rest are hard.

Bag B contains 12 balls, of which x balls are soft and the rest are hard.

Galit randomly selects a bag and takes out one random ball. Next, she returns the ball to the bag and randomly takes out a second ball from the same bag (removal with replacement).

We know that the probability that Galit took out two soft balls is $\frac{41}{200}$.

- א. Find the value of x .
א. We know Galit took out two balls of the same type. What is the probability that she took out two balls from Bag A ?

Galit performs the following procedure 4 times:

She randomly selects a bag and takes out a random ball. Next, she returns the ball to the bag and randomly takes out a second ball from the same bag (removal with replacement).

- א. What is the probability that Galit took out only hard balls in exactly two of the times?
א. What is the probability that Galit took out only hard balls two of the times and only soft balls two of the times?

Part Two – Planar Geometry and Planar Trigonometry

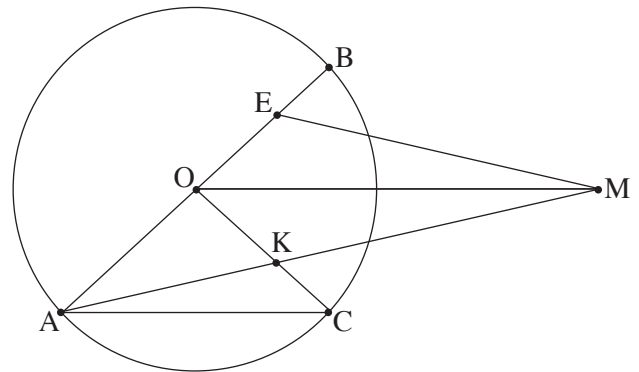
4. The figure on the right describes a circle of center O .

AB is a diameter in the circle.

Point C is located on the circle.

Point M is located outside the circle, such that Segment AM intersects Segment CO at Point K .

Point E is located on Segment BO , such that Quadrilateral $EMKO$ is a kite ($OK = OE$, $MK = ME$).



א. Prove that $BC \parallel EK$.

ב. Prove that $OM \parallel AC$.

We know that $\frac{BC}{EK} = \frac{5}{3}$.

ג. Find the value of $\frac{OM}{AC}$.

Let S be the area of Kite $EMKO$.

ד. Express the area of Triangle AOC in terms of S .

5. The figure on the right describes a trapezoid $ABCD$ ($AB \parallel DC$).

We know that the length of Base AB is equal to the length of Side BC .

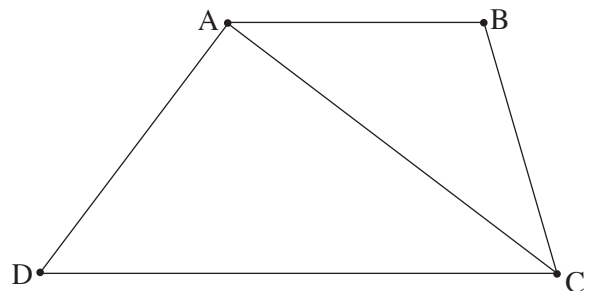
Let $AB = BC = k$, $\angle ACB = \alpha$, $0^\circ < \alpha < 90^\circ$.

א. Show that $AC = 2k \cdot \cos \alpha$.

We know that $AD = 1.2k$, $DC = 2k$.

ב. Find the value of α .

ג. Find the size of Angle ADC .



The extensions of Sides DA and CB intersect at Point E .

We know that the length of the radius of the circle inscribed in Triangle EDC is 14 .

ד. Find the value of k .

Part Three – Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions

6. Consider the function $f(x) = \frac{ax}{(x-3)^2}$, defined on the domain $x \neq 3$.
 a is a positive parameter.
- א. Answer Items (1)–(2). Express your answers in terms of a , if necessary.
- (1) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
 - (2) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
- ב. Sketch a graph of function $f(x)$.

Consider the function $g(x) = f(x) + 0.5$, defined on the domain $x \neq 3$.

We know that the graph of function $g(x)$ has exactly one common point with the x -axis.

- ג. Find the value of a .

Substitute in function $g(x)$ the value of a that you found, and answer Item 7.

Consider the function $h(x) = g(x) \cdot g'(x)$, defined on the domain $x \neq 3$.

7. (1) Give the positive and negative intervals of function $h(x)$.
- (2) Calculate the area bounded by the graph of function $h(x)$ and the x -axis on the interval $-9 \leq x \leq 0$.

7. Consider the function $f(x) = (1 + \cos x) \cdot (-1 + b \cos x)$, defined on the domain $-\pi \leq x \leq \pi$.

b is a parameter.

We know that $0 < b < 1$.

- א. Is function $f(x)$ even or odd? Explain your answer.
- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the x -axis.

We know that function $f(x)$ has an extremum [נקודת קיצון] at the point where $x = \frac{\pi}{3}$.

- ג. Find the value of b .

Substitute $b = \frac{1}{2}$ in function $f(x)$ and answer Items 7–ה.

7. (1) Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- (2) Sketch a graph of function $f(x)$.

Consider the function $h(x) = f(x) + |f(x)|$, defined on the domain $-\pi \leq x \leq \pi$.

Point A is any point located on the graph of function $h(x)$.

- ה. Is the y -coordinate of Point A positive, negative, equal to zero, or impossible to determine?
 Explain your answer.

8. The figure on the right describes the graph of function $f(x) = 2\sqrt{x^2 - 1}$.

Consider the given line $y = 3x - 3$.

- א. Find the domain [תחום הגדרה] of function $f(x)$.
- ב. Find the coordinates of the two intersection points of the graph of function $f(x)$ with the given line.

Point A is located on the graph of function $f(x)$ on the interval between the two points you found in Item ב.

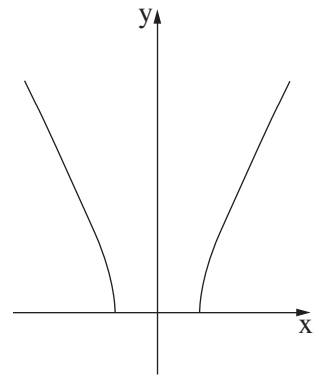
Two lines are drawn through Point A :

A line parallel to the y-axis that intersects the given line at Point B .

A line parallel to the x-axis that intersects the given line at Point C .

Let t be the x-coordinate of Point A .

- א. (1) Express the length of Segment AB in terms of t .
- (2) Express the length of Segment AC in terms of t .
- ב. Find the value of t for which the sum of the lengths of Segments AB and AC is greatest (maximum).



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך