

Mathematics

5 Points – 2nd Exam

Instructions

- א. Duration of the exam: Two and a half hours.
- ב. Exam structure and breakdown of points:
There are two parts in this exam, with a total of five questions.
Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers
Part Two: Growth and Decay, Exponential and
Logarithmic Functions
Answer three questions, at least one question from each part
 $3 \times 33\frac{1}{3} = 100$ points.

- ג. Material that may be used during the exam:
- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
 - (2) Formula sheets (attached).
 - (3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות

- א. משך הבחינה: שעתיים וחצי.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון: גאומטריה אנליטית, וקטורים,
טריגונומטריה במרחב, מספרים מרוכבים
פרק שני: גדילה ודעיכה, פונקציות מעריכיות ולוגריתמיות
יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק
 $3 \times 33\frac{1}{3} = 100$ נקודות.
- ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש.
יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider a parabola whose equation is $y^2 = 54x$.

At Point A , located on the parabola, a tangent to the parabola is drawn whose slope is $\frac{3}{4}$.

- א. Find the coordinates of Point A .

Point A is located on a circle whose center is at Point M .

We know that the tangent to the parabola is also tangent to the circle at Point A ,

and that the y -axis is tangent to the circle such that the x -coordinate of Point M is smaller than the x -coordinate of Point A .

- ב. (1) Find the equation of Line AM .
(2) Find the equation of the circle.

Point B is any point on the circle. A tangent to the circle is drawn through Point B .

Point P is located on the tangent such that $BP = 36$.

- ג. (1) Find the equation of the locus [המקום הגאומטרי] of all the points P obtained in this way.
(2) Are there two points on the locus you found whose distance from each other is 80?
Explain your answer.

2. The figure to the right shows Cuboid $ABCD A' B' C' D'$.

Let $\overrightarrow{AD} = \underline{u}$, $\overrightarrow{AB} = \underline{v}$, $\overrightarrow{AA'} = \underline{w}$.

Point E is the midpoint of Edge BB' .

א. Express $\overrightarrow{EA}$ and $\overrightarrow{EC}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.

Point M satisfies $\overrightarrow{BM} = \frac{1}{6}\underline{u} - \frac{2}{3}\underline{v} + \frac{1}{12}\underline{w}$.

ב. Express $\overrightarrow{EM}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.

We know that $\overrightarrow{EM} = a \cdot \overrightarrow{EA} + b \cdot \overrightarrow{EC}$.

a and b are parameters.

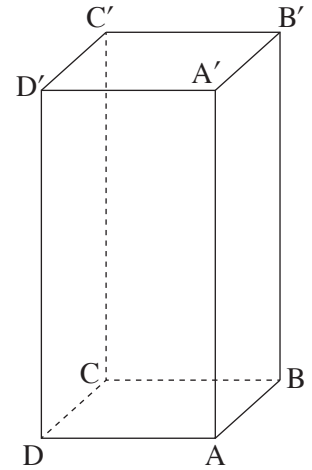
ג. (1) Find the value of a and the value of b.

(2) Is Point M located on Plane AEC? Explain your answer.

We know that $|\underline{w}| = 4$ and that Vector $\overrightarrow{BM}$ is perpendicular to Plane AEC.

ד. Find the value of $|\underline{u}|$ and the value of $|\underline{v}|$.

ה. Calculate the distance between Point B and Plane AEC.



3. Consider the equation $z^3 = -64$.

z is a complex variable.

א. Find all of the solutions to the equation.

Consider the following two complex numbers:

$$w_1 = r \cdot (\cos \alpha + i \cdot \sin \alpha), \quad w_2 = 2r \cdot (\cos 4\alpha + i \cdot \sin 4\alpha),$$

$$50^\circ < \alpha < 130^\circ.$$

Both points represented by the numbers w_1 and w_2 are located on one line passing through the origin, O, on the Gauss plane.

ב. Find both possible values of α .

We know that w_1 is one of the solutions that you found in Item א.

ג. Find the value of α and the value of r.

Consider two additional complex numbers: $w_3 = 4 \cdot i$, $w_4 = \frac{8}{i}$.

ABCD is a convex quadrilateral. Number w_1 represents Vertex A; Number w_3 represents Vertex B; Number w_2 represents Vertex C; and Number w_4 represents Vertex D.

ד. Show that the area of Quadrilateral ABCD is 36.

u is a complex number. Point E, represented by the number u, is located in the first quadrant on the bisector of Angle BOA.

$$\text{We know: } \bar{u} + \frac{1}{u} = 5.2 \cdot (\cos 285^\circ + i \cdot \sin 285^\circ), \quad |u| > 1.$$

ה. Find the complex number u.

Part Two — Growth and Decay, Exponential and Logarithmic Functions

4. Consider the function $f(x) = x^n \cdot (3 - \ln x)$, defined on the domain $[\text{תחום ההגדרה}] \ x > 0$.

The parameter n is a natural number.

- א. Find the coordinates of the intersection point of the graph of function $f(x)$ with the x -axis.

We know that the slope of the tangent to the graph of function $f(x)$ at the point where $x = 1$ is 2.

- ב. Find the value of n .

Substitute $n = 1$ in function $f(x)$, and answer Items ג–ו.

- ג. Find the coordinates of the extremum $[\text{נקודת קיצון}]$ of function $f(x)$, and determine what type of extremum it is.

- ד. Sketch a graph of function $f(x)$.

Consider the function $g(x) = \frac{1}{f(x)}$.

- ה. (1) Find the domain of function $g(x)$.
(2) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.
(3) Sketch a graph of function $g(x)$.

a is a parameter, $1 < a < 3$.

We know that the area bounded by the graph of function $g(x)$, the x -axis, and the lines

$x = e$ and $x = e^a$ is $\ln 10$.

- ו. Find the value of a .

5. Consider the function $f(x) = 4e^x - x + 9 \cdot \ln(4 - e^x)$.

- א. (1) Find the domain [תחום הגדרה] of function $f(x)$.
- (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes (if there are any).
- ב. Show that $f'(x) = \frac{-4(e^x - 1)^2}{4 - e^x}$.
- ג. (1) Find the intervals on which function $f(x)$ is increasing and decreasing (if there are any).
- (2) Find the equation of the tangent to the graph of function $f(x)$ that is parallel to the x-axis.
- ד. Sketch a graph of function $f(x)$.
- ה. (1) Find the equations of the asymptotes of the derivative function $f'(x)$ that are perpendicular to the axes.
- (2) Function $f(x)$ has only one inflection point [נקודת פיתול]. Sketch a graph of the derivative function $f'(x)$ on the interval $x < \ln 4$.

Consider the function $g(x) = f(x) + ax$, whose domain is identical to the domain of function $f(x)$.

a is a parameter.

- ו. Use the graph of the derivative function $f'(x)$ to find for which values of a function $g(x)$ has two extrema [נקודות קיצון].

Explain your answer.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך