

Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions

הוראות

א. Duration of the exam: Two hours.

א. משך הבחינה: שעותיים.

ב. Exam structure and breakdown of points:

ב. מבנה השאלון ומפתח ההערכה:

This exam has two parts, with a total of five questions.

בשאלון זה שני פרקים, ובהם חמש שאלות.

Part One – Sequences, Spatial Trigonometry

פרק ראשון – סדרות, טריגונומטריה במרחב

Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

פרק שני – גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה

Answer three questions, at least one question from each part – $3 \times 33\frac{1}{3} = 100$ points.

יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק – $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

- (1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

- (1) (מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

- (2) Formula sheets (attached).

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

- (3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

- (2) דפי נוסחאות (מצורפים).
(3) (מילון עברי–לועזי / לועזי–עברי).

ד. Special instructions:

ד. הוראות מיוחדות:

- (1) Do not copy the question, but do indicate the number of the question you are answering.

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

- (2) Start each question on a new page.

- (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.

If you use any draft paper outside the answer booklet, your exam may be disqualified.

כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Sequences, Spatial Trigonometry

Sequences

1. An ice cream shop began making chocolate ice cream for the summer vacation.

Every day, they made 3 kg more chocolate ice cream than the previous day.

The amount of chocolate ice cream made on the first and fourth day was 149 kg in total.

- א. Find how many kilograms of chocolate ice cream the shop made on the first day.

The shop made chocolate ice cream over a period of 16 days.

- ב. Find how many kilograms of chocolate ice cream the shop made in total.

The shop also made vanilla ice cream for the summer vacation.

Every day, the shop made d kg more vanilla ice cream than the previous day.

The shop made vanilla ice cream over a period of 16 days. On the last day, the shop made 115 kg of vanilla ice cream.

The total amount of vanilla ice cream the shop made was 480 kg less than the total amount of chocolate ice cream it made.

- ג. Find the value of d .

When the shop finished making the chocolate and vanilla ice cream, it started selling both flavors of ice cream.

In 5 days, the shop sold all the ice cream it had made.

Every day, the shop sold 2 times the amount of ice cream it had sold the previous day.

- ד. Find how many kilograms of ice cream the shop sold on the first day.

Spatial Trigonometry

2. The figure to the right shows a rectangular cuboid $ABCD A'B'C'D'$ whose base $ABCD$ is square.

We know that the length of the cuboid's height is 1.5 times greater than the side length of its base, and that the volume of the cuboid is 2,592.

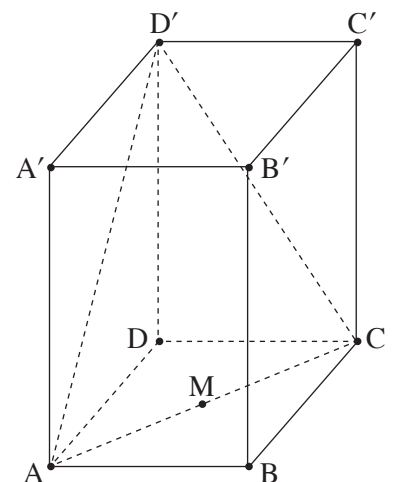
- א. (1) Find the side length of the base.
(2) Calculate the perimeter of Triangle $AD'C$.

Point M is the midpoint of Diagonal AC .

- ב. Find the size of the angle between $D'M$ and the cuboid's base, $ABCD$.

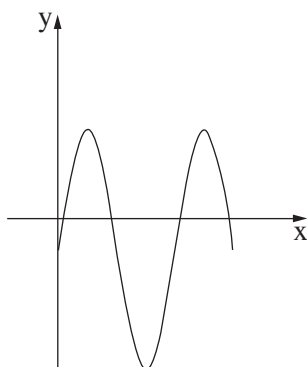
Point E is located on Edge DD' such that EM is the bisector of Angle DMD' .

- ג. Calculate the area of Triangle EDM .

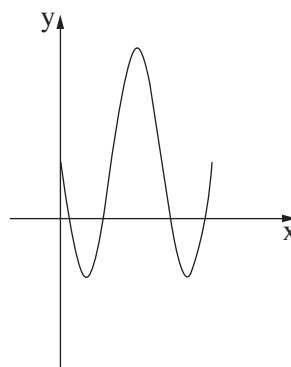


Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

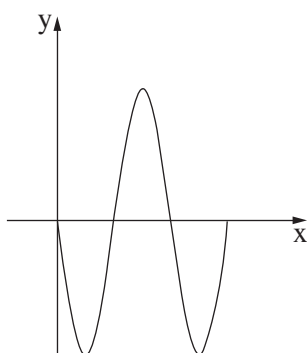
3. Function $f(x) = 4(\sin x)^2 - 1$ is defined on the domain [תחום ההגדרה] $0 \leq x \leq 1.5\pi$.
- Find the coordinates of the intersection points of the graph of function $f(x)$ with the x-axis.
 - Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
 - Sketch a graph of function $f(x)$.
 - Determine which of the graphs I–IV below describes the derivative function $f'(x)$. Explain your answer.
 - Find the area bounded by the graph of the derivative function $f'(x)$ and the x-axis on the interval $0 \leq x \leq \pi$.



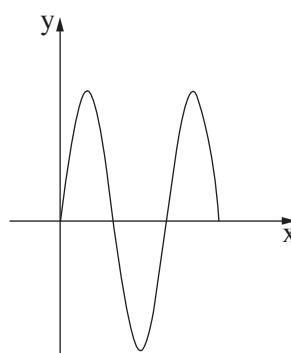
II



I



IV



III

4. Consider function $f(x) = (a - 4x) \cdot e^{2x-1}$, defined for all x .

a is a parameter.

We know that the slope of the tangent to the graph of function $f(x)$ at the point where $x = 0$ is $\frac{16}{e}$.

א. Find the value of a .

Substitute $a = 10$ in function $f(x)$, and answer Items ב-ג.

ב. (1) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.

(2) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.

ג. Sketch a graph of function $f(x)$.

Consider function $g(x) = 4 \cdot e^{2x-1}$, defined for all x .

ד. Explain why function $g(x)$ is positive for all x .

Point A is the intersection point of the graph of function $f(x)$ with the graph of function $g(x)$.

ה. Find the coordinates of Point A.

A perpendicular to the x-axis is dropped through Point A.

ו. Calculate the area bounded by the graph of function $g(x)$, the perpendicular, the x-axis, and the y-axis.

5. Consider function $f(x)$ and its derivative function $f'(x)$, both defined on the domain $x > 0$.

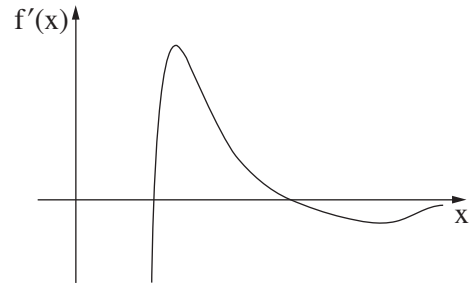
The figure on the right describes the graph of the derivative function $f'(x)$.

The figure shows all the extrema [נקודות קיצון] of the derivative function $f'(x)$ and all its intersection points with the x -axis.

- א. Consider Statements I and II, below.

Determine whether each of these statements is true or false. Explain your answers.

- I Function $f(x)$ has exactly three extrema.
II The x -coordinate of the minimum of function $f(x)$ is greater than the x -coordinate of its maximum.



We know that $f(x) = \frac{14(\ln x)^2}{x^2}$.

- א. (1) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).
(2) Find the coordinates of the extrema of function $f(x)$, and determine what type of extrema they are.
(3) Find the intervals on which function $f(x)$ is increasing and decreasing.
א. Sketch a graph of function $f(x)$.

Consider function $g(x) = f(x - 4)$.

- א. (1) What is the domain of function $g(x)$?
(2) Find the coordinates of the extrema of function $g(x)$, and determine what type of extrema they are.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך