

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד - שאלון ראשון

Instructions

הוראות

א. Duration of exam: Three and a half hours.

א. משך הבחינה: שלוש שעות וחצי.

ב. Exam structure and breakdown of points:

This exam consists of three parts, with a total of eight questions.

Part One – Statistics and Probability

Part Two – Geometry

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer five questions, at least one question from each part – $5 \times 20 = 100$ points.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

פרק ראשון – סטטיסטיקה והסתברות

פרק שני – גאומטריה

פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים,

של פונקציות רציונליות ושל פונקציות שורש

יש לענות על חמש שאלות, על שאלה אחת לפחות מכל פרק

– $20 \times 5 = 100$ נקודות.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות

במחשבון שיש בו אפשרות לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות

במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי / לועזי–עברי.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש.

יש לרשום במחברת את שלבי הפתרון, גם כאשר

החישובים מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט

ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת

הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer five of the questions 1–8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Statistics and Probability

1. A nursery called Tree and Flower grows different types of plants.

In a certain year, the lengths of the plants' stems were normally distributed, with an average length of 64 cm .

Plants with a stem length of less than 55 cm are sold to local stores.

We know that in that year 32.6% of the nursery's plants were sold to local stores.

- א. Find the standard deviation of stem lengths in that year.

Plants with a stem length of more than 80 cm are exported.

In that year, 9,540 of the nursery's plants were exported.

- ב. (1) Find the percentage of the nursery's plants that were exported.
(2) According to the normal distribution table, how many plants were neither exported nor sold to local stores?

The following year, the stem lengths of the nursery's plants were normally distributed, and the standard deviation was 25 cm .

That year, too, plants with a stem length of less than 55 cm were sold to local stores.

In that year, the nursery grew 40,000 plants in total.

4,000 of the plants were sold to local stores.

- ג. Find the average stem length in that year.

2. An advertising agency checked the relationship between advertising expenditure (variable x) and income (variable y) of 5 companies A–E in a certain month. The data are presented below.

Company	Advertising expenditure (thousands of shekels) variable x	Income (thousands of shekels) variable y
A	10	16
B	15	17
C	20	20
D	25	34
E	30	68

- א. (1) Calculate the average advertising expenditure of the 5 companies that month.
(2) Calculate the standard deviation of advertising expenditure of the 5 companies that month.

The advertising agency found that the average income of the 5 companies that month was 31 thousand shekels, and that the standard deviation of their incomes was $8\sqrt{6}$ thousand shekels.

- א. Find the correlation coefficient r .
א. (1) Find the equation of the regression line for predicting y from x .
(2) Based on the regression line, what is the predicted monthly income of a company whose monthly advertising expenditure is 24 thousand shekels?

The following month the income of each of the 5 companies grew by 15% (advertising expenditure of each of the companies remained unchanged).

- א. Below are three statements I–III.

Determine whether each statement is true or false. Explain your answers.

- I In the month when incomes increased, the standard deviation of incomes was $10\sqrt{6}$.
II In the month when incomes increased, the slope of the regression line for predicting y from x remained unchanged.
III The point (20, 35.65) is located on the regression line for predicting y from x in the month when incomes increased.

3. A certain farm grows two types of peppers: large peppers and small peppers.

The number of large peppers is 4 times greater than the number of small peppers.

One pepper is randomly selected from among all the peppers on the farm.

- א. What is the probability that the selected pepper is a small pepper?

The farm's large peppers are red or yellow,

and the small peppers are red or green.

0.6 of the large peppers are red.

0.75 of the small peppers are red.

One pepper is randomly selected from among all the peppers on the farm.

- ב. What is the probability that the selected pepper is a large yellow pepper?

- ג. (1) What is the probability that the selected pepper is a red pepper?

- (2) We know that a red pepper was selected. What is the probability that this pepper is a large pepper?

All the peppers on the farm are sent to two packing plants, A and B .

All the red peppers are sent to Plant A ; all the yellow peppers and all the green peppers are sent to Plant B .

- ד. Is the percentage of large peppers in Plant A greater or smaller than the percentage of large peppers in Plant B ? Explain your answer.

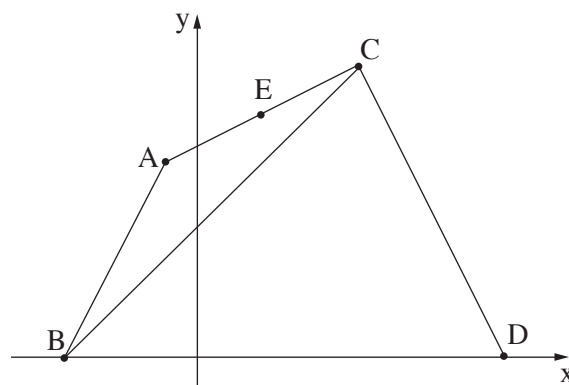
Part Two – Geometry

4. The figure on the right describes an isosceles triangle ABC ($AB = AC$).

Vertex B is located on the negative side of the x -axis.

Point D is located on the x -axis such that CD is perpendicular to AC .

We know that Point $E(4, 15)$ is the midpoint of Side AC , and that the equation of Line CD is $y = -2x + 38$.



- א. (1) Find the equation of Line AC .
- (2) Find the coordinates of Vertex C .
- ב. (1) Find the length of Side AB .
- (2) Find the coordinates of Vertex B .
- ג. Find the size of Angle ABD and the size of Angle CBD .
- ד. Calculate the area of Triangle ABC .

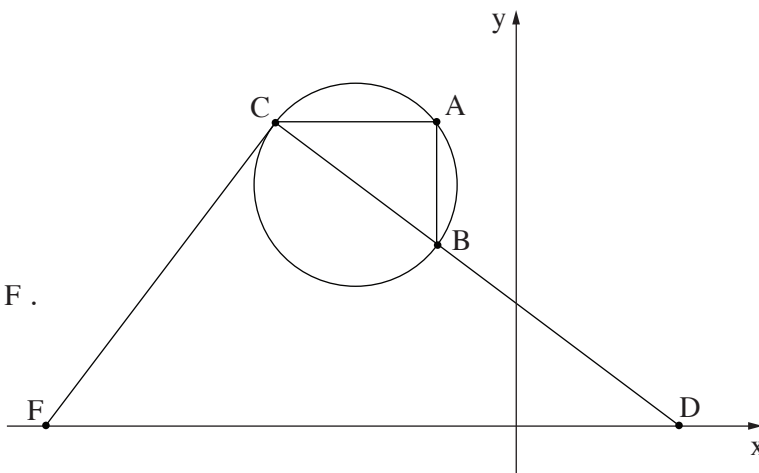
5. The figure on the right describes Triangle ABC inscribed in a circle.

Side CB is a diameter in the circle.

Side AB is parallel to the y -axis.

The extension of Side CB intersects the x -axis at Point D .

A tangent to the circle is drawn through Point C , intersecting the x -axis at Point F .



- א. (1) Explain why Side AC is parallel to the x -axis.
- (2) Prove that $\triangle ABC \sim \triangle CFD$.

We know that $D(8, 0)$, and that the equation of Tangent CF is $y = \frac{4}{3}x + 31$.

- ב. Find the length of Side FD .
- ג. (1) Find the equation of Side CD .
- (2) Find the length of Side CD .

We know that the ratio of similarity [יחס הדמיון] between Triangle ABC and Triangle CFD is $\frac{8}{25}$.

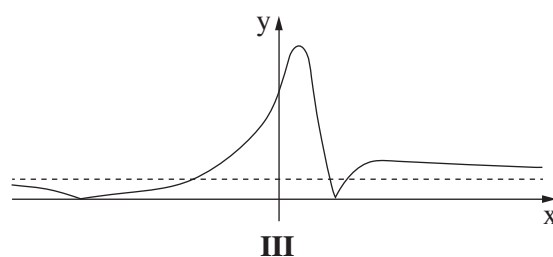
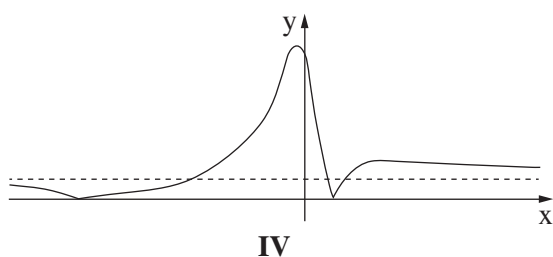
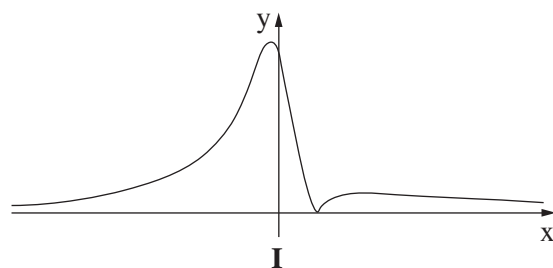
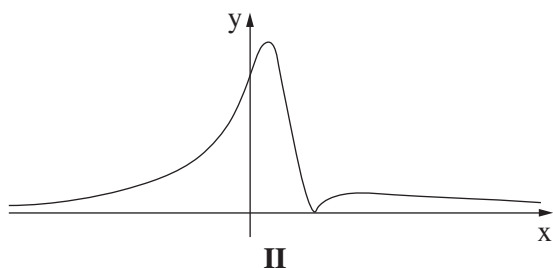
- ד. Calculate the perimeter of Triangle ABC .

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = \frac{72 - 18x}{x^2 + 9} - 1$, defined for all x .
- א. Find the equation of the asymptote of function $f(x)$ that is parallel to the x -axis.
 - ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
 - ג. (1) Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
(2) Find the intervals on which function $f(x)$ is increasing and decreasing.
 - ד. Sketch a graph of function $f(x)$.

Consider the function $g(x) = |f(x)|$, defined for all x .

- ה. (1) Find the equation of the asymptote of function $g(x)$ that is parallel to the x -axis.
(2) Determine which of the graphs I–IV below describes function $g(x)$. Explain your answer.



7. Consider the function $f(x) = (a - x) \cdot \sqrt{3x - 6}$.

a is a parameter.

א. Find the domain [תחום הגדרה] of function $f(x)$.

We know that the point $(5, 9)$ is located on the graph of function $f(x)$.

ב. Find the value of a .

Substitute $a = 8$ in function $f(x)$, and answer Items ג–ה.

ג. Find the coordinates of the intersection points of the graph of function $f(x)$ with the x -axis.

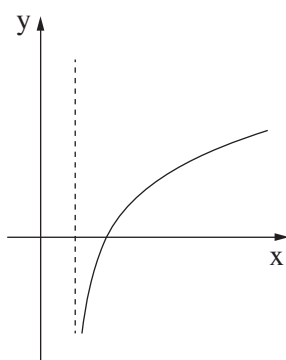
ד. Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.

ה. Sketch a graph of function $f(x)$.

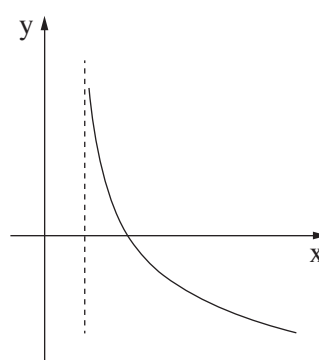
The derivative function $f'(x)$ is defined on the domain $x > 2$.

ו. (1) Determine which of the graphs I–IV at the end of this question describes the derivative function $f'(x)$. Explain your answer.

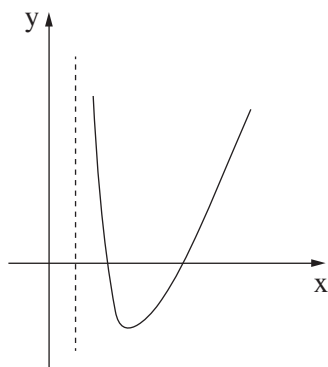
(2) Calculate the area bounded by the graph of the derivative function $f'(x)$, the x -axis, and the line $x = 20$.



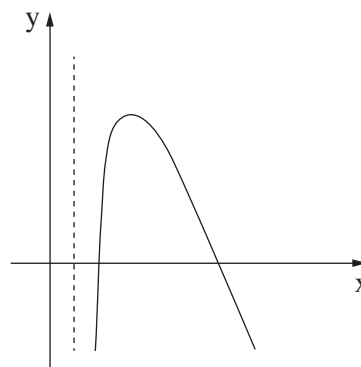
II



I



IV



III

8. The figure on the right describes a public park that is divided into two sections.

Section ABCD contains grass, and Section BFGH contains a children's playground.

Each of the sections is rectangular.

Let x denote the length of Side AB.

We know that the length of Side GF is 7 meters, and that Side BF is 2 meters longer than Side AB.

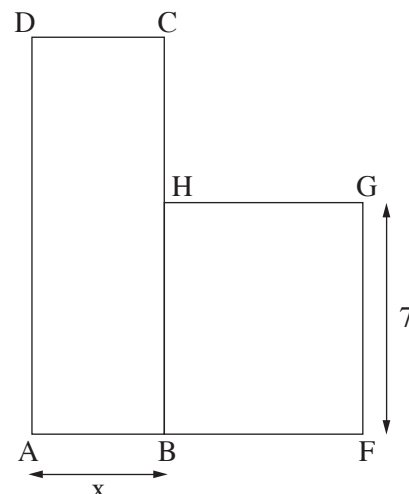
Section ABCD is surrounded by a fence 32 meters long.

א. (1) Express the length of Side AD in terms of x .

(2) Express the area of Section ABCD in terms of x .

ב. Express the ratio of the area of Section ABCD to the area of Section BFGH in terms of x .

ג. Find the value of x for which the ratio of the area of Section ABCD to the area of Section BFGH is greatest (maximum).



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך