

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Two and a half hours.

ב. Exam structure and breakdown of points:

This exam has two parts with a total of five questions.

Part One — Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers

Part Two — Growth and Decay, Exponential and
Logarithmic Functions

Answer three questions with at least one question from each
part — $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות

א. משך הבחינה: שתיים וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון — גאומטריה אנליטית, וקטורים, טריגונומטריה במרחב, מספרים מרוכבים

פרק שני — גדילה ודעיכה, פונקציות מעריכיות ולוגריתמיות
יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק

$$- 3 \times 33\frac{1}{3} = 100 \text{ נקודות}$$

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה. כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer three of the questions 1–5 with at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider a given ellipse with a canonical equation. The point $A(1, 3)$ is located on the ellipse.

The coordinates of the right focus of the ellipse are $(\sqrt{6.4}, 0)$.

- א. Find the equation of the ellipse.

Point B is the intersection point of the ellipse with the positive side of the x -axis.

A line is drawn through Point A and through the origin, O .

Point C is some point on the line AO . Point P is the midpoint of Segment BC .

- ב. Find the equation of the locus [מקום גאומטרי] of all the points P .
- ג. Find the equation of the circle that is tangent to the locus you found and to the line AO at Point O .

2. Consider the cuboid $ABCD A'B'C'D'$ in the figure to the right.

We know that the equation of the plane $A'B'C'D'$ is $x + 4y - 8z - 142 = 0$,

and that $B(0, 7, 6)$.

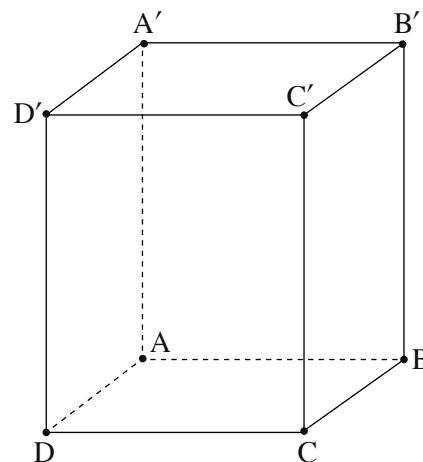
- א. Find the length of Edge BB' .

We know that a parametric form of Line AB is

$$\underline{x} = (0, 7, 6) + t(0, 2, 1).$$

The length of Edge AB is $5\sqrt{5}$, and the y -coordinate of Vertex A is negative.

- ב. Find the coordinates of Vertex A .
- ג. Find the equation of Plane $ABB'A'$.



Consider the point $M(k, 1, 3)$, k is a positive parameter.

We know that the size of the angle between Line AB and Line AM is 60° .

- ד. Find the value of k (you may leave a radical in your answer).

We know that Point M is located on Plane $DCC'D'$.

- ה. Find the volume of the cuboid.

3. א. Show that the locus [מקום גאומטרי] of all the points $z = x + iy$ on the Gauss plane which satisfy $|z - 2i| = \frac{z - \bar{z}}{2i} + 2$ is a parabola, and find the parabola's equation.

Consider a complex number $w = r(\cos \theta + i \sin \theta)$, $0^\circ < \theta < 90^\circ$, $r > 0$.

Consider a convex polygon I, whose vertices are represented by the solutions to the equation

$$z^4 = \frac{1}{r^4} \cdot (\cos(4\theta) + i \sin(4\theta)), \quad z \text{ is a complex variable.}$$

- ב. Provide a polar representation of all the numbers representing the vertices of Polygon I, expressed in terms of r and θ .

Consider Polygon II, whose vertices are represented by the numbers $-\frac{1}{w}$, $\frac{1}{w}$, $-w$, w .

- ג. Provide a polar representation of all the numbers representing the vertices of Polygon II, expressed in terms of r and θ .

We know that Polygon II is a rhombus.

- ד. Find the value of θ .

We know that the point representing w is located on the locus you found in Item א.

- ה. Find the ratio between the area of Polygon II and the area of Polygon I.

Part Two — Growth and Decay, Exponential and Logarithmic Functions

4. $f'(x)$ is the derivative function of function $f(x)$.

$g(x)$ is a function that satisfies $g(x) = e^{f(x)} \cdot f'(x)$.

Functions $f(x)$, $f'(x)$, and $g(x)$ are defined on the domain [תחום הגדרה] $x \neq 0$.

We know that function $f(x)$ is decreasing on the intervals $x > 0$ and $x < 0$,
and that $f'(x) \neq 0$ on its entire domain.

- א. List the negative and positive intervals of function $g(x)$ (if there are any). Explain your answer.

We know that $f(x) = \frac{1}{x}$.

- ב. (1) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.
(2) Find the coordinates of the extremum [נקודת קיצון] of function $g(x)$, and determine what type of extremum it is.
ג. Sketch a graph of function $g(x)$.

a is a parameter greater than 1.

We know that the area bounded by the graph of function $g(x)$, the x -axis, and the lines

$x = \frac{1}{\ln a}$ and $x = \frac{1}{\ln(4a)}$ is 5.

- ד. Find the value of a .

5. Consider the function $f(x) = \frac{1}{x} \left(a + \frac{1}{(\ln x)^2} \right)$. a is a positive parameter.

א. Find the domain [תחום הגדרה] of function $f(x)$.

We know that function $f(x)$ has only one extremum [נקודת קיצון] at a point where $x = \frac{1}{e}$.

ב. Find the value of a .

Substitute $a=1$ in function $f(x)$, and answer Items ג-ו.

$g(x)$ is a function whose derivative satisfies $g'(x) = f(x)$.

The domain of function $g(x)$ is identical to the domain of function $f(x)$.

We know that function $g(x)$ has an inflection point [נקודת פיתול] that is located on the x -axis.

ג. Find a function $g(x)$ that satisfies these conditions.

Answer Items ד-ו for the function $g(x)$ that you found.

ד. (1) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes (if there are any).

(2) Find the coordinates of the intersection points of the graph of function $g(x)$ with the x -axis.

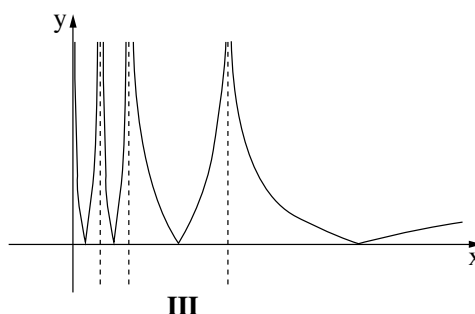
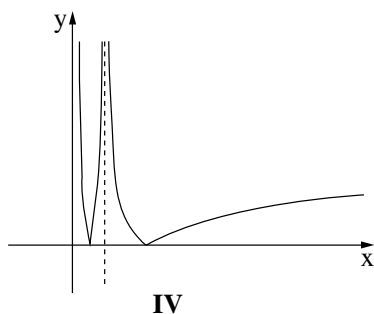
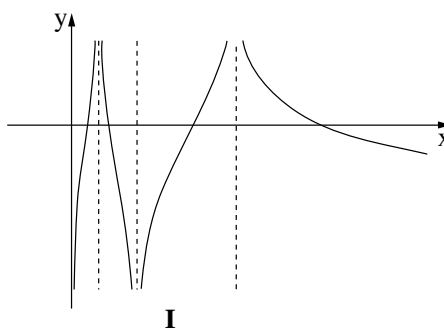
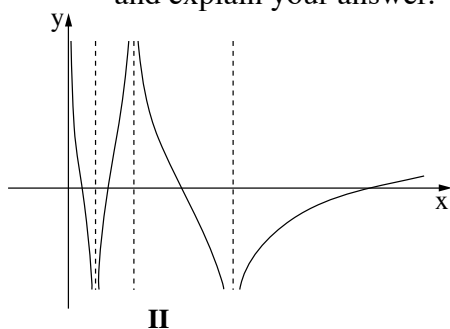
(3) Find the intervals on which function $g(x)$ is increasing and decreasing (if there are any).

ה. Sketch a graph of function $g(x)$.

Consider the function $h(x) = \ln |g(x)|$.

ו. (1) Find the domain of function $h(x)$.

(2) One of the graphs I-IV shown below describes function $h(x)$. Determine which one it is and explain your answer.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך