



Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions

הוראות

- א. Duration of the exam: Two hours. משך הבחינה: שתיים.
- ב. Exam structure and breakdown of points: מבנה השאלון ומפתח ההערכה:
This exam consists of two parts, with a total of five questions. בשאלון זה שני פרקים, ובהם חמש שאלות.
Part One – Sequences, Spatial Geometry, and Growth and Decay פרק ראשון – סדרות, גאומטריה במרחב וגדילה ודעיכה
Part Two – Differential and Integral Calculus of Exponential and Logarithmic Functions פרק שני – חשבון דיפרנציאלי ואינטגרלי של פונקציות מעריכיות ולוגריתמיות
יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק — $3 \times 33 \frac{1}{3} = 100$ נקודות.
- ג. Material that may be used during the exam: חומר עזר מותר בשימוש:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
Use of a graphing calculator or programming options may lead to the disqualification of your exam. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) Formula sheets (attached). (2) דפי נוסחאות (מצורפים).
(3) A Hebrew–foreign language / foreign language–Hebrew dictionary. (3) מילון עברי–לועזי / לועזי–עברי.
- ד. Special instructions: הוראות מיוחדות:
(1) Do not copy the question, but do indicate the number of the question you are answering. (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) Start each question on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
Explain all of your work, including calculations, in detail and in a clear and organized fashion. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
Lack of detail may lower your score or lead to the disqualification of your exam. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טייטה" at the top of each draft page.
If you use any draft paper outside the answer booklet your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Sequences, Spatial Geometry, and Growth and Decay

1. A certain town held a book fair that ran for 7 days.

The number of books sold each day at the book fair was 2 times greater than the number of books sold the previous day.

We know that a total of 420 books were sold during the first three days of the fair.

- א. Find how many books were sold on the first day of the fair.

Each book at the fair was sold for 50 shekels.

- ב. Find the total income from book sales during the 7 days of the fair.

When the book fair ended its director decided to donate half of the total income from the fair.

The director spread the donation over several months.

In the first month he donated 4,315 shekels, then in each subsequent month he donated 315 shekels more than in the previous month.

- ג. Find over how many months the fair director spread the donation.
ד. Find the total amount that the fair director donated in the two middle months.

2. The figure on the right describes a right prism $ABCD A'B'C'D'$ whose base $ABCD$ is a parallelogram.

Point M is the midpoint of Edge $C'D'$,
and Point P satisfies $\overrightarrow{AP} = \frac{3}{4}\overrightarrow{AA'}$.

Let $\overrightarrow{AB} = \underline{u}$, $\overrightarrow{AD} = \underline{v}$, $\overrightarrow{AA'} = \underline{w}$.

- א. Express the vectors $\overrightarrow{CM}$ and $\overrightarrow{PM}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.

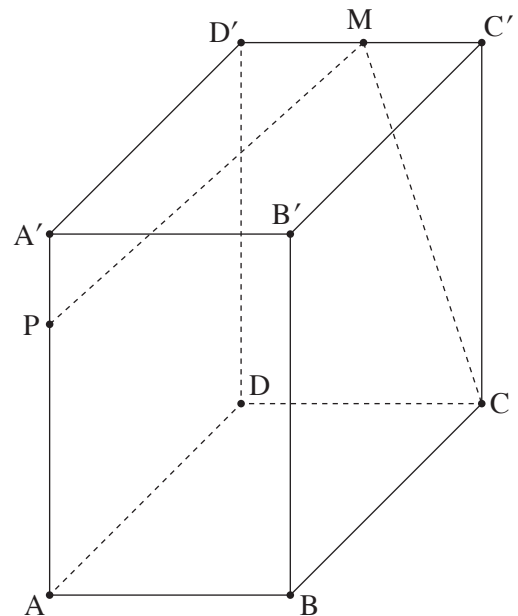
We know that $|\underline{u}| = 8$, $|\underline{v}| = 10$, $\angle DAB = 60^\circ$.

- ב. Find the value of each of the following scalar products:

$$\underline{u} \cdot \underline{v}, \underline{v} \cdot \underline{w}, \underline{u} \cdot \underline{w}.$$

We know that the prism's volume is $480\sqrt{3}$.

- ג. Find the value of $|\underline{w}|$.
ד. Prove that $\overrightarrow{PM}$ is perpendicular to $\overrightarrow{CM}$.
ה. (1) Show that $|\overrightarrow{PM}| = \sqrt{165}$.
(2) Calculate the area of Triangle PMC .



3. Ya'el made two kinds of jam, strawberry jam and plum jam.

She measured the initial temperature of the jams she had made—at the same time—and immediately placed them in the refrigerator.

She measured the jams' temperature in degrees Celsius.

The initial temperature of the strawberry jam was higher than the initial temperature of the plum jam.

The temperature of each of the jams decreased by a fixed percentage per minute (each jam by a different percentage).

We know that a certain amount of time after the moment the jams were placed in the refrigerator, the plum jam's temperature was higher than the strawberry jam's temperature.

א. Did the temperature of the strawberry jam decrease by a higher percent per minute than the temperature of the plum jam? Explain your answer.

We know that the initial temperature of the strawberry jam was 42 degrees, and that the jam's temperature decreased by 4% a minute.

ב. Find how much time after the moment the jams were placed in the refrigerator the strawberry jam reached 15 degrees.

6 minutes after the moment the jams were placed in the refrigerator, the temperature of the plum jam was 88% of its initial temperature.

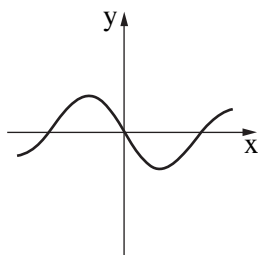
ג. Find by what percentage the temperature of the plum jam decreased every minute.

10 minutes after the moment the jams were placed in the refrigerator, the temperature of the plum jam was 31 degrees.

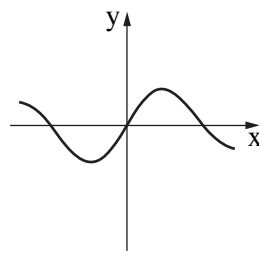
ד. Find how much time after the moment the jams were placed in the refrigerator the jams reached the same temperature.

Part Two – Differential and Integral Calculus of Exponential and Logarithmic Functions

4. The function $f(x)$ and its derivative function $f'(x)$ are defined for all x .
Function $f(x)$ has only one minimum and two maxima.
One of the graphs I and II, below, describes the derivative function $f'(x)$.
The figures show all the intersection points of each graph with the x -axis.



II



I

- א. Determine which of the graphs describes the derivative function $f'(x)$. Explain your answer.

We know that $f(x) = \frac{6x^2}{e^{x^2+2}}$.

- ב. Find the coordinates of the intersection point of the graph of function $f(x)$ with the x -axis.
- ג. Prove that function $f(x)$ is even.
- ד. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- ה. Sketch a graph of function $f(x)$.
- ו. Calculate the area bounded by the graph of the derivative function $f'(x)$ and the x -axis.

5. Consider the function $f(x) = ax \cdot (2 - \ln x)$, a is a non-zero parameter.

א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the coordinates of the intersection point of the graph of function $f(x)$ with the x -axis.

We know that at the point where $x = e^3$ the slope of the tangent to the graph of function $f(x)$ is -8 .

ב. Find the value of a .

Substitute $a = 4$ in function $f(x)$, and answer Items ג-ד.

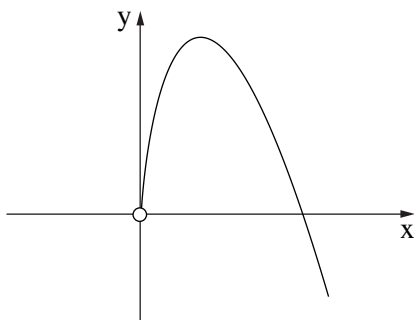
ג. (1) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.

(2) Determine which of the graphs I–IV at the end of this question describes function $f(x)$. Explain your answer.

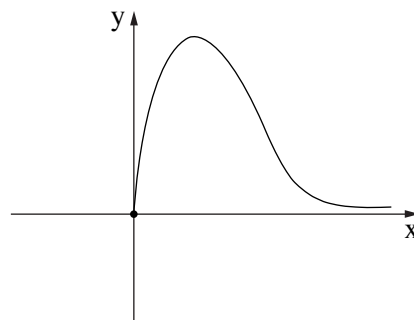
Consider the function $g(x) = -2f(x) + 37$.

ד. (1) Find the coordinates of the extremum of function $g(x)$, and determine what type of extremum it is.

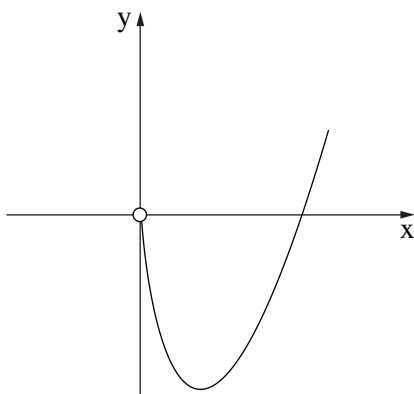
(2) Does the graph of function $f(x)$ intersect the graph of function $g(x)$? Explain your answer.



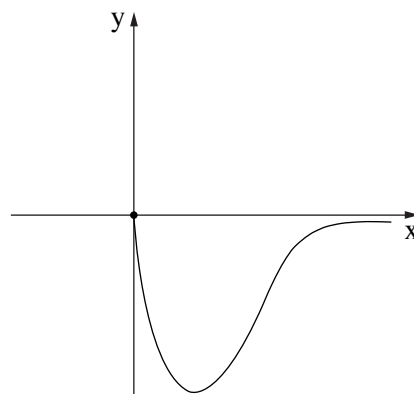
II



I



IV



III

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך