

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד – שאלון ראשון

Instructions

הוראות

א. Duration of the exam: Three and a half hours.

א. משך הבחינה: שלוש שעות וחצי.

ב. Exam structure and breakdown of points:

This exam consists of three parts, with a total of eight questions.

Part One – Statistics and Probability

Part Two – Geometry

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer five questions, at least one question from each part – $5 \times 20 = 100$ points.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

פרק ראשון – סטטיסטיקה והסתברות

פרק שני – גאומטריה

פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

יש לענות על חמש שאלות, על שאלה אחת לפחות מכל פרק
— $5 \times 20 = 100$ נקודות.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות לתכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי / לועזי–עברי.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer five of the questions 1–8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Statistics and Probability

1. The psychometric test scores in a particular year were normally distributed, and the average score was 550 .

That year, Limor took the psychometric test.

Limor's score on the test was 686 , and her standard score was 1.6 .

- א. Find the standard deviation of scores in the test.

The cut-off score is the lowest score that must be achieved on the test to be eligible for admission to a particular department.

The table below shows the cut-off score needed to be eligible for admission to the literature department and the cut-off score needed to be eligible for admission to the psychology department at a certain university.

	Literature department	Psychology department
Cut-off score	584	669

Yael took the test, too, that year. We know that 57.9 % of the examinees scored lower than Yael.

- א. Is Yael eligible for admission to the literature department? Explain your answer.
- ב. (1) Find the percentage of examinees eligible for admission to the psychology department.
(2) Find the percentage of examinees eligible for admission to the literature department, but ineligible for admission to the psychology department.

We know that 8,448 examinees are eligible for admission to the literature department, but are ineligible for admission to the psychology department.

- ג. According to the normal distribution table, how many examinees are eligible for admission to the psychology department?

2. A nutritionist examined the relationship between the maximum temperature in Jerusalem on a given day (Variable x) and how much water Yossi, a resident of Jerusalem, drank that day (Variable y). The table below presents the data collected by the nutritionist over 6 days.

Maximum temperature (degrees Celsius) – Variable x	Yossi's water consumption (liters) – Variable y
20	1.6
22	1.7
24	2.4
26	2.7
28	2.9
30	3.1

- א. (1) Find the average maximum temperature in the 6 days examined.
(2) Find the standard deviation of maximum temperature in the 6 days examined.

The nutritionist calculated that Yossi's average water consumption in the 6 days examined was 2.4 liters, and the standard deviation of his water consumption was $\frac{7\sqrt{6}}{30}$ liters.

- ב. Find r , the correlation coefficient [מקדם המתאם] between the two variables.
ג. Find the equation of the regression line for predicting y according to x .

The nutritionist multiplied each of the numbers for Yossi's water consumption (Variable y) by 3, in order to represent the water consumption of Yossi's entire family.

- ד. For each of the measures I and II below, determine whether its value increased, decreased, or remained unchanged, after the numbers were multiplied. Explain your answers.
I The average of Variable y .
II The standard deviation of Variable y .
ה. Find the slope of the regression line for predicting y according to x after the numbers were multiplied. Explain your answer.

3. In a certain town, families are encouraged to recycle things like glass bottles and plastic packaging. The families are also encouraged to volunteer in the town's institutions.

A survey conducted among all families in the town found that some families recycle, while the rest of the families do not recycle.

Similarly, some families in the town volunteer, and the rest do not volunteer.

A family from this town is selected at random. The probability that this family recycles is greater by 0.44 than the probability that this family does not recycle.

$\frac{5}{9}$ of the families that recycle are families that volunteer.

- א. (1) Find the percentage of families in this town that recycle.
(2) Find the percentage of families in this town that recycle and also volunteer.

The number of families in this town that recycle and also volunteer is 5 times greater than the number of families that do not recycle and also do not volunteer.

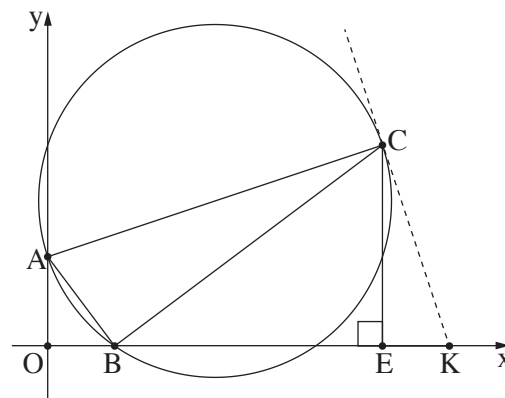
- ב. A family from this town is selected at random. What is the probability that this family volunteers?
ג. A family from this town that volunteers is selected at random. What is the probability that this family does not recycle?

There are 750 families in the town.

- ד. Find how many families in this town recycle or volunteer.

Part Two – Geometry

4. In the figure to the right Triangle ABC is inscribed in a circle. AC is a diameter of the circle. Vertex A is located on the y-axis and Vertex B is located on the x-axis. A perpendicular is drawn through Point C to the x-axis intersecting it at Point E. Point O is the origin.



- א. (1) Prove that $\angle ABO = \angle BCE$.
(2) Prove that $\triangle AOB \sim \triangle BEC$.

We know that the similarity ratio between Triangle BEC and Triangle AOB is 3, and that $A(0, 12)$, $B(9, 0)$.

- א. (1) Find the length of Side BE.
(2) Find the coordinates of Vertex C.

A tangent to the circle is drawn at Point C.

- א. Find the equation of the tangent.

Point K is the intersection point of the tangent with the x-axis.

- א. Find the area of Quadrilateral ABKC.

5. The figure to the right shows Kite ABCD ($AB = CB$, $AD = CD$).

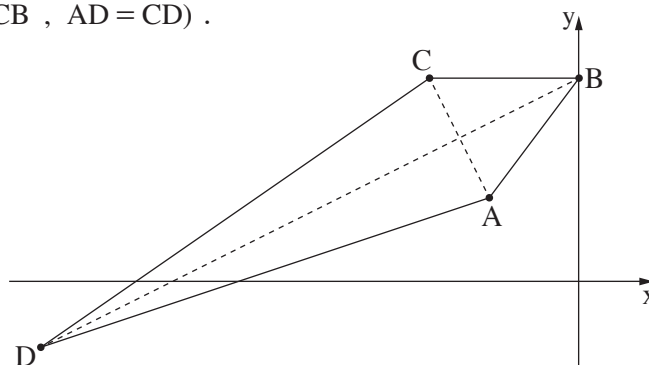
Vertex C is located in the second quadrant.

Side BC is parallel to the x-axis.

We know: $A(-3, 2)$, $B(0, 6)$.

- א. (1) Find the length of Side AB.
(2) Find the coordinates of Vertex C.

- א. Find the equation of Diagonal DB.



We know that the equation of Side AD is $y = \frac{1}{3}x + 3$.

- א. Find the coordinates of Vertex D.

Point M is the intersection point of the kite's diagonals.

- א. Find the size of Angle ADM.

Point E is the midpoint of Side DC.

- א. Calculate the area of Triangle ADE.

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = \frac{a}{(x-4)^2}$, defined on the domain $x \neq 4$.

a is a positive parameter.

We know that at the point where $x = 6$, the slope of the tangent to the graph of function $f(x)$ is -3 .

א. Find the value of a .

Substitute $a = 12$ and answer items ב–ה.

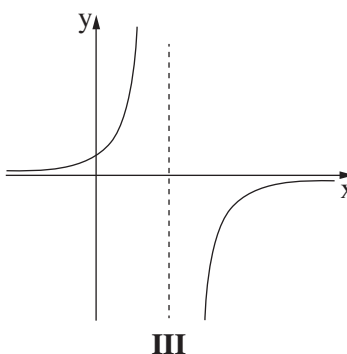
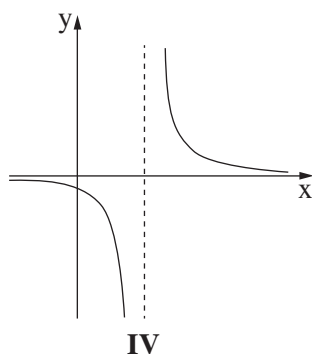
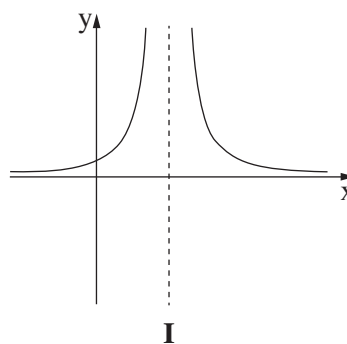
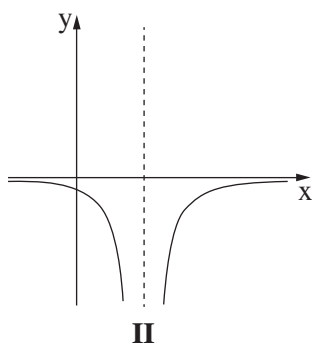
- ב. (1) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).
(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
(3) Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.
ג. Sketch a graph of function $f(x)$.

The domain of the derivative function $f'(x)$ is the same as the domain of function $f(x)$.

- ד. (1) Determine which of the graphs I–IV at the end of the question describes the graph of the derivative function $f'(x)$.
(2) Calculate the area bounded by the graph of the derivative function $f'(x)$, the line $x = 3$, the x -axis, and the y -axis.

Consider the function $g(x) = f'(x) + 2$.

- ה. By how much is the area bounded by the graph of function $g(x)$, the line $x = 3$, the x -axis, and the y -axis, larger than the area that you found in Item ד(2)? Explain your answer.



7. Consider the function $f(x) = x^2 \cdot \sqrt{7-x}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the coordinates of the intersection points of the graph of function $f(x)$ with the x -axis.
- (3) Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- ב. Sketch a graph of function $f(x)$.

Consider the function $g(x) = f(x+a)$, a is a parameter.

We know that function $g(x)$ has a local maximum [נקודת מקסימום פנימית] at a point where $x = 8.6$.

- ג. (1) Find the value of a . Explain your answer.
- (2) What is the domain of function $g(x)$?

8. The figure below describes the graph of the function $f(x) = \frac{6x-12}{x-8}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

Point A is the intersection point of the graph of function $f(x)$ with the x -axis.

- ב. Find the coordinates of Point A.

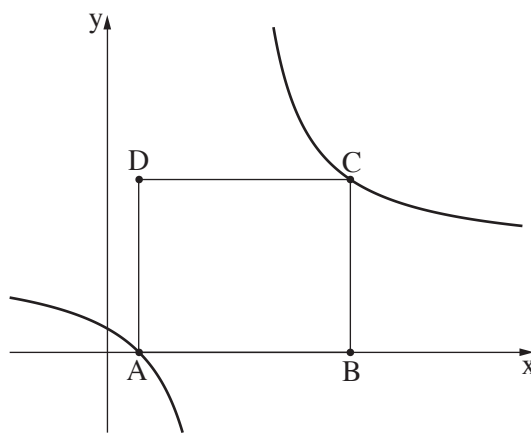
Point C is located on the graph of function $f(x)$ on the interval $x > 8$.

From Point C a perpendicular is drawn to the x -axis intersecting it at Point B.

Point D is located in the first quadrant such that ABCD is a rectangle.

Let t denote the x -coordinate of Point C.

- ג. Express the perimeter of Rectangle ABCD in terms of t .
- ד. Find the value of t for which the perimeter of Rectangle ABCD is minimum (smallest).
- ה. Is it possible that the perimeter of Rectangle ABCD is 50? Explain your answer.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך