

Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

5 Points – 2nd Exam

Instructions

- א. Duration of the exam: Two hours and forty-five minutes.
ב. Exam structure and breakdown of points:
There are two parts in this exam, with a total of five questions.
Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers
Part Two: Growth and Decay, Exponential and
Logarithmic Functions
Answer three questions, at least one question from each part
 $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות

- א. משך הבחינה: שתיים וארבעים וחמש דקות.
ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון: גאומטריה אנליטית, וקטורים,
טריגונומטריה במרחב, מספרים מרוכבים
פרק שני: גדילה ודעיכה, פונקציות מעריכיות ולוגריתמיות
יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק
— $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider an ellipse whose equation is $\frac{x^2}{25k^2} + \frac{y^2}{16k^2} = 1$. k is a positive parameter.

Point F is the right focus of the ellipse.

Consider a circle whose center is at Point F , and which passes through the intersection point of the ellipse with the positive side of the x -axis.

א. Express the equation of the circle in terms of k .

Point B is the intersection point of the ellipse with the positive side of the y -axis.

Point D is the intersection point of Segment BF with the circle.

ב. (1) Find the ratio of Segment BD to Segment DF .

(2) Express the coordinates of Point D in terms of k .

ג. Express the equation of the tangent to the circle at Point D in terms of k .

We construct a square whose side length is 15.

One of the sides of the square lies on the tangent whose equation you found in Item ג.

Point B is located inside the square.

ד. Express in terms of k the equation of the line on which the side of the square parallel to the tangent lies.

We know that Point B is the intersection point of the diagonals of the square.

ה. Find the value of k .

2. The figure on the right is a right prism $AOBA'O'B'$ whose base AOB is a right triangle, $\angle AOB = 90^\circ$.

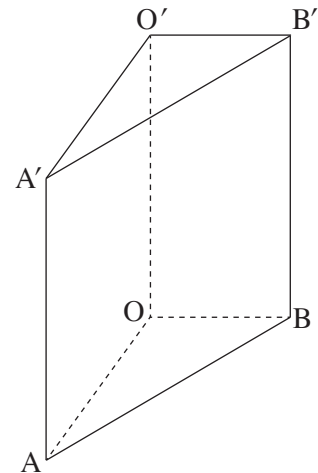
Point K is the midpoint of Edge AB .

Point E is located on Segment $O'B$ such that $\overrightarrow{O'E} = \frac{1}{3} \cdot \overrightarrow{O'B}$.

Point N is located on Segment AE such that $\overrightarrow{AN} = \frac{3}{4} \cdot \overrightarrow{AE}$.

Let $\overrightarrow{OA} = \underline{u}$, $\overrightarrow{OB} = \underline{v}$, $\overrightarrow{OO'} = \underline{w}$.

- א. (1) Express $\overrightarrow{O'N}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.
(2) Prove that the points O' , N , K are on one line, and find the ratio of $O'K$ to $O'N$.



We know that $O(0, 0, 0)$, $A(4, 0, 0)$, $E(0, 1, 6)$;

Vertex B is located on the positive side of the y -axis, and Vertex O' is located on the positive side of the z -axis.

- ב. Find the coordinates of points N and B .
ג. Find the equation of Plane $A'KB'$.
ד. (1) Find the distance between Point N and Plane $A'KB'$.
(2) Find the volume of Pyramid $NA'KB'$.

3. The product of two complex numbers z_1 and z_2 is: $-19 + 4i$.

$$z_1 = 2 + 3i$$

- א. Find the number z_2 .

Consider the equation $w^3 = z_1 + z_2$, w is a complex variable.

- ב. Find the solutions to the equation.

Consider a complex number $z = 2(\cos \theta + i \sin \theta)$, $0^\circ \leq \theta < 360^\circ$.

We know that the numbers z and z^3 are located on one line that passes through the origin.

- ג. Find the four possibilities for the number z .

The solutions to the equation that you found in Item ב and the four numbers you found in Item ג represent all the vertices of a convex [קמור] polygon on the Gauss plane.

- ד. (1) Sketch the polygon on a coordinate system.
(2) Calculate the area of the polygon.

Consider a complex number $3(\cos \alpha + i \sin \alpha)$, $0^\circ \leq \alpha < 360^\circ$.

Each of the numbers representing the vertices of the polygon is multiplied by the above complex number, so that a new polygon is formed.

- ה. Find by what factor the area of the new polygon is greater than the area of the polygon you found in Item ד.

Part Two — Growth and Decay, Exponential and Logarithmic Functions

4. Consider the function $f(x) = \frac{e^x + x}{e^x - x} + b$, defined for all x .
 b is a parameter.

א. Answer Sub-Items (1)–(3). If necessary, express your answers in terms of b .

- (1) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
- (2) Find the coordinates of the intersection point of the graph of function $f(x)$ with the y -axis.
- (3) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$ and determine what type of extremum it is.

We know that function $f(x)$ has a horizontal asymptote whose equation is $y = 1.5$.

- ב. Find the value of b (find both possibilities).

Substitute in function $f(x)$ the smaller value of b that you found, and answer Items ג–ה.

- ג. Sketch a graph of function $f(x)$.

We know that the intersection point of the graph of function $f(x)$ with the y -axis is one of the two inflection points [נקודות פיתול] of function $f(x)$.

The derivative function $f'(x)$ is defined for all x .

- ד. (1) Find the equations of the asymptotes of the derivative function $f'(x)$ that are perpendicular to the axes (if there are any).
- (2) Sketch a graph of the derivative function $f'(x)$.

- ה. Consider the proposition $\int_0^1 (f(x) - 1.5) dx > \frac{1}{e-1}$.

Is this proposition correct? Explain your answer.

5. Consider the function $f(x) = \frac{x \cdot \ln(x)}{1 + \ln(x)}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x-axis.
- (3) Find the coordinates of the intersection point of the graph of function $f(x)$ with the x-axis.
- ב. Find the intervals on which the function $f(x)$ is increasing and decreasing (if there are any).
- ג. Sketch a graph of function $f(x)$.

Consider the function $g(x) = \frac{1 + \ln(x)}{x \cdot \ln(x)}$.

- ד. (1) Find the domain of function $g(x)$.
- (2) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.
- (3) Sketch a graph of function $g(x)$.

Consider the expression $\int_{e^k}^{e^{2k}} \left(\frac{1}{x \cdot \ln(x)} + \frac{1}{x} \right) dx$, k is a parameter greater than 1.

- ה. (1) Determine which expression, I or II, is equal to the value of the given expression. Explain your answer.
I $k + \ln(k)$ **II** $k + \ln 2$
- (2) Calculate the area bounded by the graph of function $g(x)$, the x-axis, and the lines $x = e^3$ and $x = e^6$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך