

Note: This exam has special instructions. Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions

הוראות

א. Duration of the exam: Two and a quarter hours

א. משך הבחינה: שתיים ורבע.

ב. Exam structure and breakdown of points:

ב. מבנה השאלון ומפתח ההערכה:

This exam has two parts, with a total of five questions.

בשאלון זה שני פרקים, ובהם חמש שאלות.

Part One – Sequences, Spatial Trigonometry

פרק ראשון – סדרות, טריגונומטרייה במרחב

Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

פרק שני – גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה

Answer three questions, at least one question from each part – $3 \times 33\frac{1}{3} = 100$ points.

יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק – $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) Formula sheets (attached).

(2) דפי נוסחאות (מצורפים).

(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

(3) מילון עברי–לועזי / לועזי–עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each question on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

Lack of detail may lower your score or lead to the disqualification of your exam.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.

If you use any draft paper outside the answer booklet, your exam may be disqualified.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your booklet will be graded.

Part One – Sequences, Spatial Trigonometry

Sequences

1. Michal deposited money into a savings account every month. The amount she deposited each month was greater than the amount she deposited the previous month by a constant number.

In the first and second months, Michal deposited a total sum of 2,100 shekels.

Michal's deposit in the fifth month was 3 times greater than her deposit in the first month.

א. Find how much Michal deposited in the first month.

ב. Find after how many months the total sum that Michal had deposited was 22,680 shekels.

In the 12th month, instead of depositing money in her savings account, Michal decided to buy a refrigerator that cost the same amount as she would have deposited that month.

Michal paid for the refrigerator in four installments. Each installment was 3 times greater than the previous one.

ג. Find how much the first installment was.

Spatial Trigonometry

2. The figure to the right shows a right quadrilateral pyramid SABCD whose base ABCD is a rectangle.

SO is the pyramid height.

We know that $BC = 9$, $AB = 10$,

the perimeter of Triangle SAC is 46.

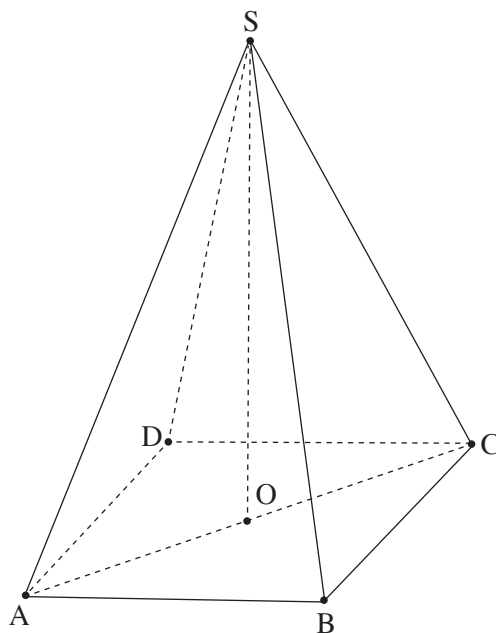
- א. (1) Find the length of the base diagonal.
(2) Find the size of the angle between a side edge and the pyramid base.
ב. Find the volume of the pyramid.

Point E is located on the pyramid height SO.

Segments are drawn from Point E to the vertices of the base ABCD, forming a new pyramid EABCD.

We know that the volume of Pyramid EABCD is smaller by 200 than the volume of Pyramid SABCD.

- ג. (1) Find the length of Segment EO.
(2) Find the size of Angle SAE.



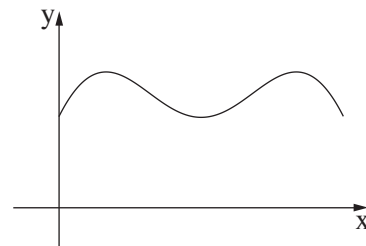
Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

3. The figure on the right shows the graph of the function $f(x) = \cos(2x) + 2 \sin x$ on the domain $0 \leq x \leq \pi$.

א. Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

A tangent to the graph of function $f(x)$ was drawn at the local minimum.

- ב. Find the equation of the tangent.
ג. Find the area bounded by the graph of function $f(x)$ and the tangent.



Consider function $g(x) = f(x) + c$ defined on the domain $0 \leq x \leq \pi$.

- ד. Give an example of a value of c for which the graph of function $g(x)$ intersects the x -axis at exactly four points.

4. Consider the function $f(x) = \frac{e^{2x}}{e^x - 4}$.

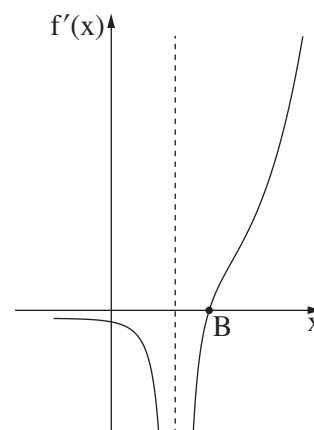
- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
(2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x -axis.
ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).

The figure on the right describes the graph of the derivative function $f'(x)$.

Function $f(x)$ and the derivative function $f'(x)$ are defined in the same domain.

The graph of the derivative function $f'(x)$ intersects the x -axis at only one point, Point B.

- ג. Find the x -coordinate of Point B.
ד. (1) Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.
(2) Sketch a graph of function $f(x)$.
ה. Calculate the area bounded by the graph of the derivative function $f'(x)$, the x -axis, and the line $x = \ln 2$.



5. Consider the function $f(x) = x^2 \cdot (-2 + \ln x)$.

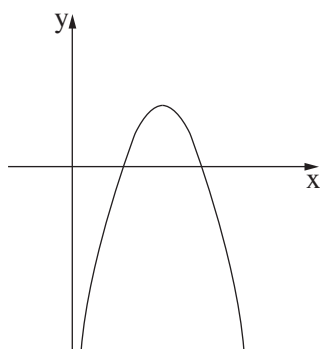
- א. Find the domain [תחום ההגדרה] of function $f(x)$.
- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).
- ג. Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
- ד. Determine which of the graphs I–IV at the end of the question describes function $f(x)$. Explain your answer.

Consider the function $g(x) = a \cdot f(x)$. a is a parameter that is not 0.

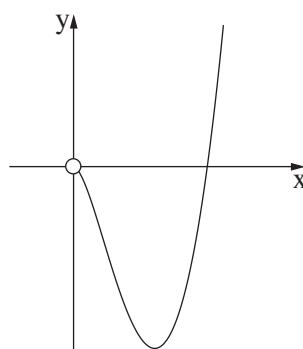
Function $g(x)$ and function $f(x)$ are defined in the same domain.

We know that the y-coordinate of the extremum of function $g(x)$ is $0.6 \cdot e^3$.

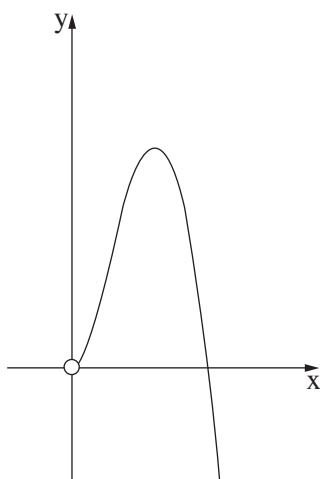
- ה. (1) Find the value of a .
- (2) What type of extremum does function $g(x)$ have? Explain your answer.
- (3) Determine which of the graphs I–IV below describes function $g(x)$. Explain your answer.



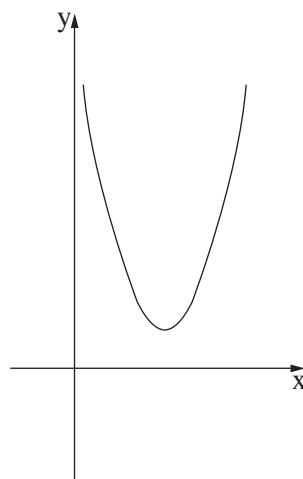
II



I



IV



III

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך