

Note: There are special instructions in this exam. Please answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 1st Exam

Instructions

- א. Duration of the exam: Four hours.
- ב. Exam structure and breakdown of points:
This exam has three parts with a total of eight questions.
Part One: Algebra, Analytical Geometry and Probability
Part Two: Planar Geometry and Planar Trigonometry
Part Three: Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer **five** questions, at least one question from each part
 $- 5 \times 20 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language/
foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.

If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

4 יחידות לימוד – שאלון ראשון

הוראות

- א. משך הבחינה: ארבע שעות.
ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון – אלגברה, גאומטריה אנליטית, והסתברות
פרק שני – גאומטריה וטריגונומטריה במישור
פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

יש לענות על **חמש** שאלות, על שאלה אחת לפחות מכל פרק
 $- 5 \times 20 = 100$ נקודות.

- ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי / לועזי–עברי.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להסביר כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל פעולותיכם, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer five of the questions 1-8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra, Analytical Geometry and Probability

1. At 8:00, Noam set off to walk on a 16 km long trail.

He walked 4 km at a constant speed and then took a break for 6 minutes.

After the break, Noam continued walking at a speed that was 2 km/h faster than his initial walking speed.

Noam reached the end of the trail at 11:50.

א. Find Noam's initial walking speed.

ב. If Noam had not stopped at all, and after walking 4 km had walked at a speed that was 3 km/h faster than his initial speed, at what time would he have reached the end of the trail?

2. The figure on the right describes a square ABCD.

Vertex A lies on the positive side of the y-axis, and Vertex B lies on the negative side of the x-axis.

The diagonals of the square intersect at Point E.

We know that the equation of Diagonal AC is $y = -\frac{1}{4}x + 12$ and the side length of the square is $\sqrt{544}$.

א. Find the coordinates of Vertices A and B.

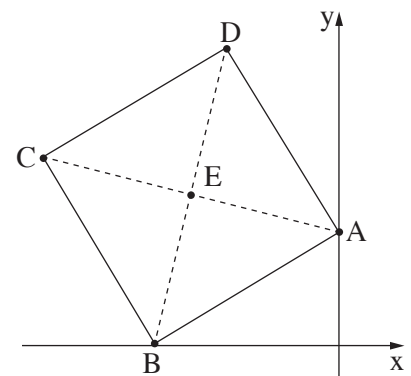
ב. (1) Find the equation of Diagonal BD.

(2) Find the coordinates of Point E.

ג. Find the equation of the circle that circumscribes Square ABCD.

The circle whose equation you found in Item ג intersects the y-axis at an additional point, P.

ד. Calculate the perimeter of Quadrilateral AEDP.



3. Two drawers, A and B, contain two types of fruit: apples and pears.

Drawer A contains 5 apples and 11 pears.

Drawer B contains 4 apples and 6 pears.

One fruit is taken at random from Drawer A and then one fruit is taken at random from Drawer B.

א. What is the probability of taking two different types of fruit?

ב. (1) What is the probability of taking at least one apple?

(2) If we know that at least one apple was taken, what is the probability that two apples were taken?

Each fruit was returned to the drawer from which it was taken.

All the fruit in the two drawers were transferred to Drawer C, which was empty.

After that, x apples were added to Drawer C.

Two fruits were then taken at random from Drawer C one after another (without replacement).

We know that the probability of taking two pears is $\frac{8}{33}$.

ג. Find the value of x .

Part Two — Planar Geometry and Planar Trigonometry

4. Triangle ABC is inscribed in a circle whose center is O .

AB is a diameter of the circle.

A tangent to the circle is drawn through Point C .

Point E is located on the tangent such that Segment AE is perpendicular to Segment CE (see figure).

- א. Prove that $\triangle ABC \sim \triangle ACE$.

We know that the length of the radius of the circle is 5,

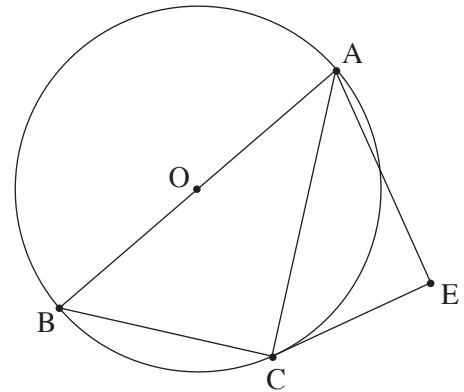
$AE = 6.4$.

- ב. Find the length of Segment AC .

The extension of Radius CO intersects the circle at an additional point, P .

- ג. Prove that Quadrilateral $ACBP$ is a rectangle.

- ד. Find by what factor the area of Rectangle $ACBP$ is greater than the area of Triangle ACE .



5. The figure on the right describes Triangle ABC .

BM is the bisector of Angle ABC .

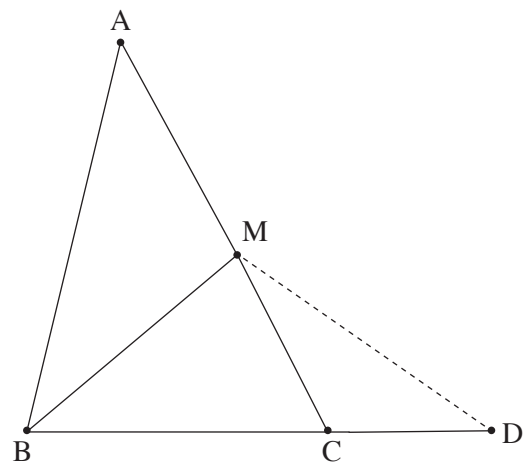
We know that: $AB = 14$, $BM = 10$, $AM = 8.5$.

- א. (1) Find the size of Angle BAM .
(2) Find the size of Angle ABM .

Point D is located on the extension of Side BC .

We know that the area of Triangle BMD is 50.

- ב. Find the length of Segment BD .
ג. Find the length of the radius of the circle that circumscribes Triangle MCD .



Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = \frac{9x}{(x+3)^2}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
- (3) Find the coordinates of the intersection point of the graph of function $f(x)$ with the x-axis.
- (4) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
- ב. Sketch a graph of function $f(x)$.

The line $y = k$ and the graph of function $f(x)$ meet at exactly one point, $k > 0$.

- ג. Find the value of k .

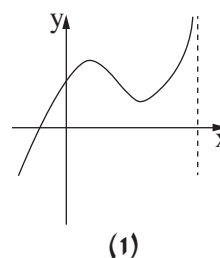
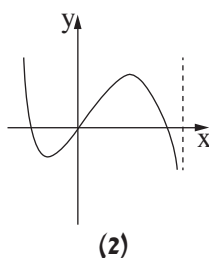
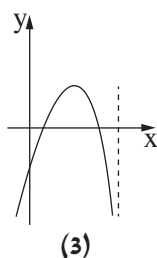
Consider the function $g(x) = f(x) + 1$.

- ד. (1) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.
- (2) For the value of k that you found, how many points are common to the line $y = k$ and the graph of function $g(x)$? Explain your answer.

7. Function $f(x)$ is defined on the domain $x \leq 1.5$,
and its derivative function $f'(x)$ is defined on the domain $x < 1.5$.

Function $f(x)$ has exactly two local extrema [נקודות קיצון פנימיות].

Below are three graphs, (1)–(3). One of them describes the derivative function $f'(x)$.



- א. Determine which of the graphs (1)–(3) describes the derivative function $f'(x)$. Explain your answer.

We know that $f(x) = (x^2 + 1) \cdot \sqrt{3 - 2x}$.

- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
- ג. (1) Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- (2) Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.
- ד. Sketch a graph of function $f(x)$.
- ה. Find the area bounded by the graph of the derivative function $f'(x)$ and the x-axis.

8. The figure on the right describes the graph of the function $f(x) = \frac{x-3}{x-4}$ on the interval $x > 4$.

Point O is the origin.

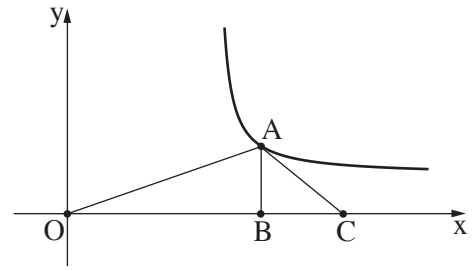
Point A is located on the graph of function $f(x)$ on the interval $x > 4$.

A perpendicular to the x-axis is dropped from Point A, intersecting the x-axis at Point B.

Point C is located on the x-axis, to the right of Point B.

We know that $OB = 2 \cdot BC$.

Let t denote the x-coordinate of Point A.



- א. (1) Express the length of Segment OC in terms of t .
 (2) Express the area of Triangle AOC in terms of t .
 ב. Find the value of t for which the area of Triangle AOC is minimum.

Good Luck!

All rights reserved by the State of Israel.
 Copying or publication without the consent of the
 Ministry of Education is prohibited.

בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך