

Note: This exam has special instructions. Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions

הוראות

- א. Duration of the exam: Two and a quarter hours.
- ב. Exam structure and breakdown of points:
This exam has two parts, with a total of five questions.
Part One – Sequences, Spatial Geometry, and Growth and Decay
Part Two – Differential and Integral Calculus of Exponential and Logarithmic Functions
Answer three questions, at least one from each part
 $3 \times 33\frac{1}{3} = 100$ points.
- ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.
- ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

- א. משך הבחינה: שתיים ורבע.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון – סדרות, גאומטריה במרחב וגדילה ודעיכה
פרק שני – חשבון דיפרנציאלי ואינטגרלי של פונקציות מעריכיות ולוגריתמיות
יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק
 $3 \times 33\frac{1}{3} = 100$ נקודות.
- ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי / לועזי-עברי.
- ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.
If you use any draft paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Sequences, Spatial Geometry, and Growth and Decay

1. Michal deposited money into a savings account every month. The amount she deposited each month was greater than the amount she deposited the previous month by a constant number.

In the first and second months, Michal deposited a total sum of 2,100 shekels.

Michal's deposit in the fifth month was 3 times greater than her deposit in the first month.

א. Find how much Michal deposited in the first month.

ב. Find after how many months the total sum that Michal had deposited was 22,680 shekels.

In the 12th month, instead of depositing money in her savings account, Michal decided to buy a refrigerator that cost the same amount as she would have deposited that month.

Michal paid for the refrigerator in four installments. Each installment was 3 times greater than the previous one.

ג. Find how much the first installment was.

2. The figure on the right shows a triangular pyramid $SABC$.

Let $\overrightarrow{AB} = \underline{u}$, $\overrightarrow{AC} = \underline{v}$, $\overrightarrow{AS} = \underline{w}$.

Point E is located on Edge SB and Point F is located on Edge SC .

We know that $\overrightarrow{SE} = \frac{3}{4}\overrightarrow{SB}$, $\overrightarrow{SF} = \frac{3}{4}\overrightarrow{SC}$.

א. Express the vectors $\overrightarrow{BC}$, $\overrightarrow{SF}$, and $\overrightarrow{SE}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.

ב. (1) Express the vector $\overrightarrow{EF}$ in terms of $\underline{u}$ and $\underline{v}$.

(2) Explain why EF is parallel to BC .

We know that $B(1, -1, 3)$, $C(13, 15, 3)$.

ג. Find the algebraic representation of the vector $\overrightarrow{BC}$ and the vector $\overrightarrow{EF}$.

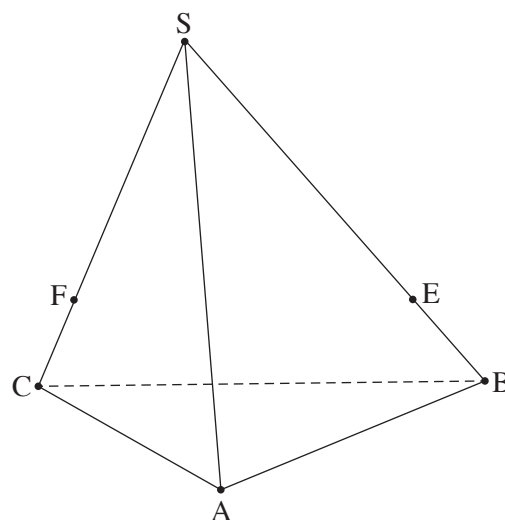
We know that $E(5, 1, 6)$.

ד. Find the coordinates of Point F .

We know that $A(4, 12, 3)$.

ה. (1) Find the size of Angle EAF .

(2) Calculate the area of Triangle EAF .

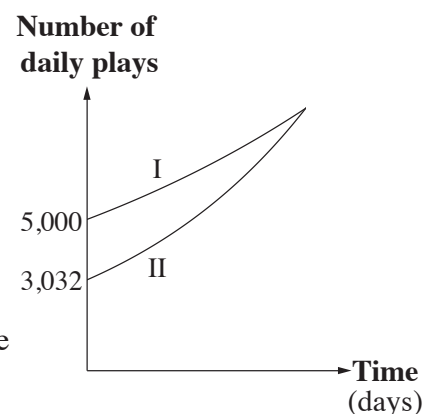


3. Two songs, A and B, were uploaded to the same social media platform on the same day.

For Song A, the number of daily plays [האזנות] increased by a fixed percentage every day.

For Song B, the number of daily plays increased every day by a percentage that was 2 times greater than the percentage by which the number of Song A's daily plays increased every day.

The two graphs, I and II, on the right describe the number of daily plays for each song as a function of time since the songs were uploaded.



- א. Determine which of the graphs, I or II, represents Song A.

Explain your answer.

We know that 5 days after the day the songs were uploaded, the number of daily plays for Song A was 7,350.

- ב. Find by what percentage the number of daily plays for Song A increased every day.
- ג. Find how much greater the number of daily plays for Song A was than the number of daily plays for Song B, 5 days after the day the songs were uploaded.
- ד. Find how much time after the day the songs were uploaded the number of daily plays for Song A was equal to the number of daily plays for Song B.

When the number of daily plays for Song A was equal to the number of daily plays for Song B, the number of daily plays for Song A began to drop.

Every day the number of daily plays for Song A decreased by the same percentage as it had increased previously.

- ה. Find how long after the day the songs were uploaded the number of daily plays for Song A was 2,450.

Part Two – Differential and Integral Calculus of Exponential and Logarithmic Functions

4. Consider the function $f(x) = \frac{e^{2x}}{e^x - 4}$.

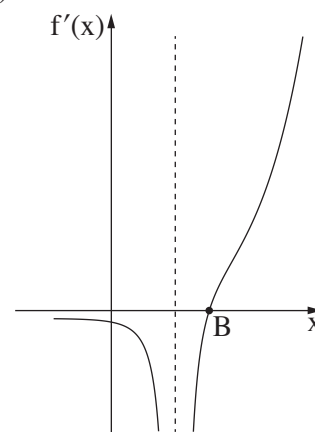
- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x-axis.
- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).

The figure on the right describes the graph of the derivative function $f'(x)$.

Function $f(x)$ and the derivative function $f'(x)$ are defined on the same domain.

The graph of the derivative function $f'(x)$ intersects the x-axis at only one point, Point B.

- ג. Find the x-coordinate of Point B.
- ד. (1) Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.
- (2) Sketch a graph of function $f(x)$.
- ה. Calculate the area bounded by the graph of the derivative function $f'(x)$, the x-axis, and the line $x = \ln 12$.



5. Consider the function $f(x) = x^2 \cdot (-2 + \ln x)$.

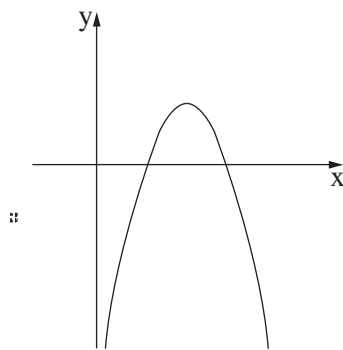
- א. Find the domain [תחום ההגדרה] of function $f(x)$.
- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).
- ג. Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
- ד. Determine which of the graphs I–IV at the end of the question describes function $f(x)$. Explain your answer.

Consider the function $g(x) = a \cdot f(x)$. a is a parameter that is not 0.

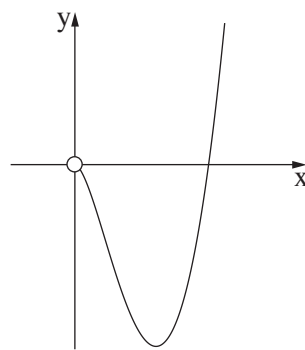
Function $g(x)$ and function $f(x)$ are defined in the same domain.

We know that the y-coordinate of the extremum of function $g(x)$ is $0.6 \cdot e^3$.

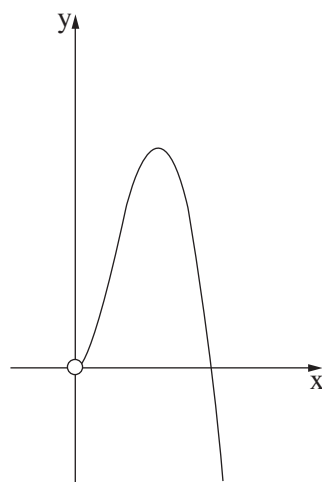
- ה. (1) Find the value of a .
- (2) What type of extremum does function $g(x)$ have? Explain your answer.
- (3) Determine which of the graphs I–IV below describes function $g(x)$. Explain your answer.



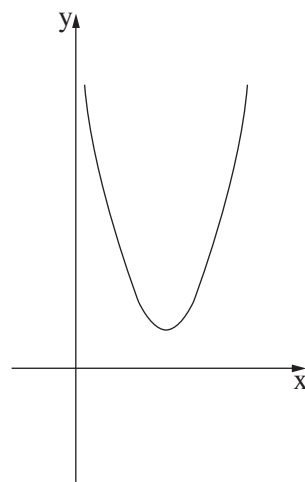
II



I



IV



III

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך