

תוכנית חדשה

Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד – שאלון ראשון

Instructions

הוראות

א. Duration of the exam: Four hours.

א. משך הבחינה: ארבע שעות.

ב. Exam structure and breakdown of points:

ב. מבנה השאלון ומפתח ההערכה:

This exam consists of three parts, with a total of eight questions.

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

Part One – Statistics and Probability

פרק ראשון – סטטיסטיקה והסתברות

Part Two – Geometry

פרק שני – גאומטריה

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

יש לענות על חמש שאלות, על שאלה אחת לפחות מכל פרק

— $20 \times 5 = 100$ נקודות.

Answer five questions, at least one question from each part — $5 \times 20 = 100$ points.

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי / לועזי-עברי.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language / foreign language-Hebrew dictionary.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

Lack of detail may lower your score or lead to the disqualification of your exam.

חסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.

כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer five of the questions 1–8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Statistics and Probability

1. The grades in a university entrance exam are normally distributed, and the standard deviation is 8 .
93.3% of examinees received a grade lower than 94 .

א. Find the average grade in this exam.

44% of examinees—those whose grades were highest—were accepted to the university.

- ב. Was an examinee with a grade of 84 accepted to the university? Explain your answer.

Some of the examinees are defined as outstanding, and the rest are not.

Examinees with a grade higher than 92 are defined as outstanding.

- ג. Find what percentage of examinees were accepted to the university and were not outstanding.

2,171 examinees who were accepted to the university, were not outstanding.

- ד. According to the normal distribution table, how many examinees in total took the exam?

Examinees who were not accepted to the university could take an additional exam.

The grades in this second exam were also normally distributed and the average grade was 80 .

Nadav took both exams and received a grade of 74 in each exam. Nadav's standard score in both exams was the same.

- ה. Find the standard deviation in the second exam.

2. Yoav investigated the linear relationship between his running distance (x variable) and his running speed (y variable) on 6 different routes, A–F .

The table below presents the data.

Route	Running distance (km) – x variable	Running speed (km/h) – y variable
A	2	12
B	4.5	11
C	6	11.5
D	7	?
E	8	8.5
F	8.5	7

- א. Find Yoav's average running distance for the 6 routes.
 ב. Show that the standard deviation of Yoav's running distance on the 6 routes is $S_x = \sqrt{\frac{59}{12}}$.

Yoav found that his average running speed on the 6 routes was the same as his running speed on Route D .

- ג. Calculate Yoav's running speed on Route D .

Consider the numbers: 0.95 , 0.24 , -1 , and -0.86 . One of them is the correlation coefficient r .

- ד. Determine which of the numbers is the correlation coefficient r , and explain your answer.

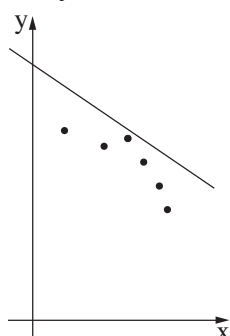
Yoav calculated the standard deviation of his running speed on the 6 routes and found that it was

$$S_y = \sqrt{\frac{37}{12}} .$$

- ה. (1) Find the equation of the regression line for predicting y according to x .
 (2) Based on the regression line, what is Yoav's predicted running speed for a distance of 4 km?

Below are three scatter diagrams, I–III. The dots on each diagram describe the data for Yoav's running speed on the 6 routes.

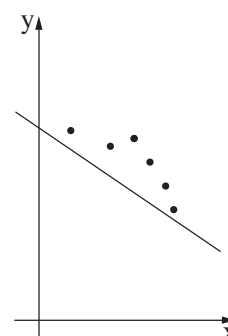
- ו. Determine which of the diagrams I–III describes the regression line that you found in Item ה. Explain your answer.



III



II



I

3. Two drawers, A and B, contain two types of fruit: apples and pears.

Drawer A contains 5 apples and 11 pears.

Drawer B contains 4 apples and 6 pears.

One fruit is taken at random from Drawer A and then one fruit is taken at random from Drawer B.

א. What is the probability of taking two different types of fruit?

ב. (1) What is the probability of taking at least one apple?

(2) If we know that at least one apple was taken, what is the probability that two apples were taken?

Each fruit was returned to the drawer from which it was taken.

All the fruit in the two drawers were transferred to Drawer C, which was empty.

After that, x apples were added to Drawer C.

Two fruits were then taken at random from Drawer C one after another (without replacement).

We know that the probability of taking two pears is $\frac{8}{33}$.

ג. Find the value of x .

Part Two – Geometry

4. The figure on the right shows Rhombus $ABCD$.

The diagonals of the rhombus meet at Point E .

We know that:

the equation of Side AB is $y = -1.75x + 11.75$,

the x -coordinate of Vertex A is -3 ,

and the coordinates of Vertex C are $(-11, 1)$.

א. (1) Find the coordinates of Point E .

(2) Find the slope of Diagonal AC .

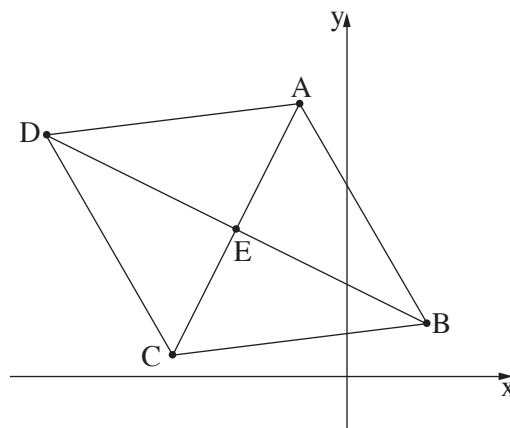
ב. (1) Find the equation of Diagonal BD .

(2) Find the coordinates of Vertex B .

ג. Find the size of Angle ABE .

Point G is located on Segment DE , such that the size of Angle GAB is 97° .

ד. Find the length of Segment GE .



5. Consider a circle that intersects the positive side of the y -axis at Points A and D , as shown in the figure.

BD and AC are chords in the circle which intersect at Point E .

We know that Chord BD is parallel to the x -axis.

א. (1) Explain why AB is a diameter of the circle.

(2) Prove that $\angle CBD = \angle DAC$.

We know that $BE = AE$.

ב. Prove that $\triangle CBE \cong \triangle DAE$.

We know that $E(-4.5, 6)$, $AE = 7.5$.

ג. Find the coordinates of Points A and B .

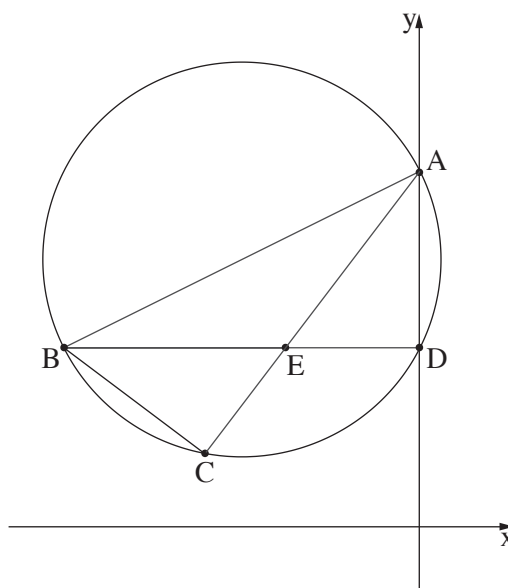
Point M is the center of the circle.

ד. Find the equation of the circle.

ה. (1) What is the length of Side BC ?

Explain your answer.

(2) Calculate the perimeter of Quadrilateral $BCEM$.



Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = \frac{9x}{(x+3)^2}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
- (3) Find the coordinates of the intersection point of the graph of function $f(x)$ with the x -axis.
- (4) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
- ב. Sketch a graph of function $f(x)$.

The line $y = k$ and the graph of function $f(x)$ meet at exactly one point, $k > 0$.

- ג. Find the value of k .

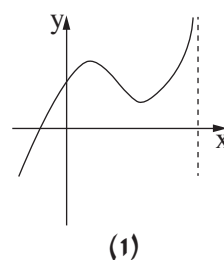
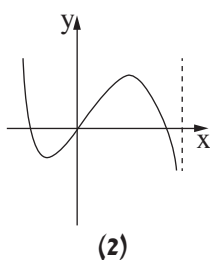
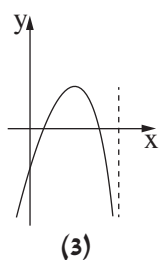
Consider the function $g(x) = f(x) + 1$.

- ד. (1) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.
- (2) For the value of k that you found, how many points are common to the line $y = k$ and the graph of function $g(x)$? Explain your answer.

7. Function $f(x)$ is defined on the domain $x \leq 1.5$,
 and its derivative function $f'(x)$ is defined on the domain $x < 1.5$.

Function $f(x)$ has exactly two local extrema [נקודות קיצון פנימיות].

Below are three graphs, (1)–(3). One of them describes the derivative function $f'(x)$.



- א. Determine which of the graphs (1)–(3) describes the derivative function $f'(x)$. Explain your answer.

We know that $f(x) = (x^2 + 1) \cdot \sqrt{3 - 2x}$.

- ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
- ג. (1) Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- (2) Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.
- ד. Sketch a graph of function $f(x)$.
- ה. Find the area bounded by the graph of the derivative function $f'(x)$ and the x -axis.

8. The figure on the right describes the graph of the function $f(x) = \frac{x-3}{x-4}$ on the interval $x > 4$.

Point O is the origin.

Point A is located on the graph of function $f(x)$

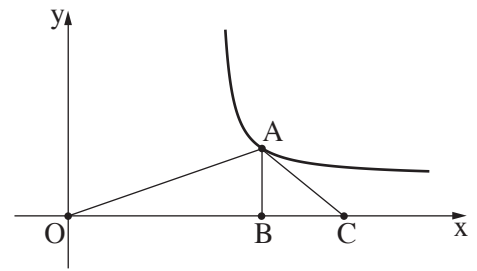
on the interval $x > 4$.

From Point A, a perpendicular is dropped to the x-axis, intersecting the x-axis at Point B.

Point C is located on the x-axis, to the right of Point B.

We know that $OB = 2 \cdot BC$.

Let t denote the x-coordinate of Point A.



- א. (1) Express the length of Segment OC in terms of t .
- (2) Express the area of Triangle AOC in terms of t .
- ב. Find the value of t for which the area of Triangle AOC is minimum.