

Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

5 Points – 2nd Exam

Instructions

- א. Duration of the exam: Two hours and forty-five minutes.
ב. Exam structure and breakdown of points:
There are two parts in this exam, with a total of five questions.
Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers
Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions
Answer three questions, at least one question from each part
 $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

5 יחידות לימוד – שאלון שני

הוראות

- א. משך הבחינה: שתיים וארבעים וחמש דקות.
ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון: גאומטריה אנליטית, וקטורים,
טריגונומטריה במרחב, מספרים מרוכבים
פרק שני: גדילה ודעיכה, פונקציות חזקה,
פונקציות מעריכיות ולוגריתמיות
יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק
 $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider Circle I whose equation is $x^2 - 6x + y^2 + t = 0$. t is a parameter smaller than 9.

א. Express the radius of the circle in terms of t .

The line $y = x$ is tangent to the circle.

ב. Find the value of t .

Circle I is shifted leftward such that Circle II is formed with its center located at the origin, O.

A is any point on Circle II such that $-1.5 \leq y_A \leq 1.5$.

Chord AB is drawn through Point A, perpendicular to the y-axis.

Point P is located on Chord AB such that $AB = 2 \cdot PO$.

ג. Show that the locus [המקום הגאומטרי] of all points P obtained in this way are located on an ellipse, and find the equation of the ellipse.

Point K is the right focus [מוקד] of the ellipse that you found in Item ג, and Point F is the left focus of the ellipse.

C is any point on the ellipse such that Triangle CKF is formed.

ד. Calculate the perimeter of Triangle CKF.

ה. Is there a point C for which the area of Triangle CKF is 3? Explain your answer.

2. Consider Pyramid $SABCD$ whose base $ABCD$ is a parallelogram.

SA is the height of the pyramid.

Point M is the midpoint of Diagonal BD .

Let $\overrightarrow{AB} = \underline{u}$, $\overrightarrow{AD} = \underline{v}$, $\overrightarrow{AS} = \underline{w}$.

א. Express the vector $\overrightarrow{SM}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.

We know that $\overrightarrow{SM}$ is perpendicular to $\overrightarrow{DB}$.

ב. Prove that $|\underline{u}| = |\underline{v}|$.

We know: $A(0, 0, 0)$, $B(0, 5, 0)$, $S(0, 0, 6)$.

$D(3, p, 0)$, $p > 0$.

ג. Find the coordinates of Points D and C .

A plane is drawn through Diagonal AC , parallel to Edge SD .

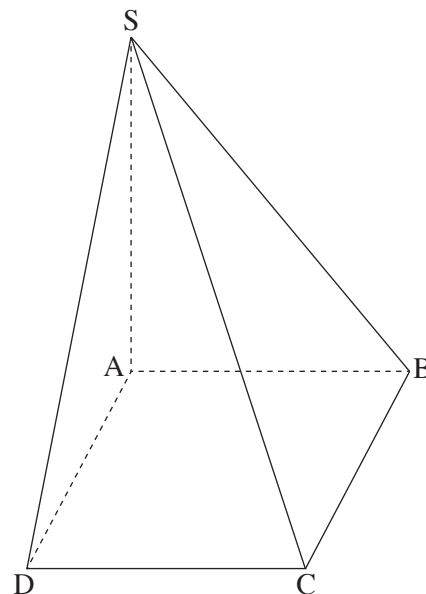
The plane intersects Edge SB at Point K .

ד. Find the equation of the plane.

ה. Below are two statements I–II. Determine whether each statement is true or false. Explain your answers.

I Line KM is parallel to Line SD .

II Any line that is perpendicular to Plane ACK is perpendicular to Line SD .



3. א. Express, in terms of x and y , the equation of the locus [המקום הגאומטרי] of all points $z = x + iy$ on the Gauss plane that satisfy $|z| = |z - (2 + 2i)|$.

z is a complex variable.

Consider the equation $z^2 - z(2 + 2i) + 4i = 0$.

A and B are points represented by the two solutions to the equation.

ב. Show that Points A and B are located on the locus that you found in Item א.

Points C and D are located on the axes such that $ABCD$ is a square.

Square $ABCD$ is rotated counter-clockwise around the origin at an angle of α ($0^\circ < \alpha < 90^\circ$).

All eight points (Points A , B , C , D , and the four points obtained after the rotation) are represented by solutions to the equation $z^n = b$, in which n is a natural number, and b is a real number.

ג. (1) Find the value of b and the value of α for $n = 8$.

(2) Find a possible value of n for $\alpha = 15^\circ$, and the corresponding value of b .

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the function $f(x) = \frac{e^x}{a - e^x}$ where a is a non-zero parameter.

א. Answer Sub-Items (1)–(3) for $a > 0$ and for $a < 0$.

If necessary, express your answers in terms of a .

(1) Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the y-axis.

(3) Find the intervals on which function $f(x)$ is increasing and decreasing (if there are any).

ב. Determine which of the graphs I–IV at the end of this question describes function $f(x)$ for $a = 3$, and which of the graphs describes function $f(x)$ for $a = -3$. Explain your answers.

Substitute $a = 3$ in function $f(x)$, and answer Items ג–ה.

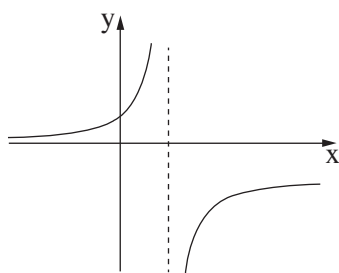
Consider the function $g(x) = \frac{1}{f(x) + 2}$.

ג. (1) Find the domain of function $g(x)$.

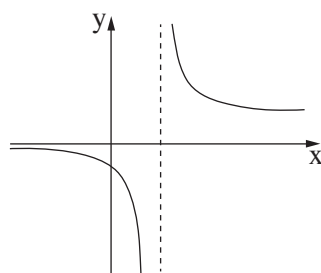
(2) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.

ד. Sketch a graph of function $g(x)$.

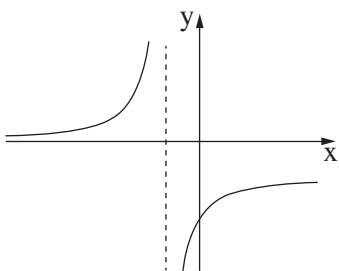
ה. Consider the statement $\int_{-4}^{-2} g(x) dx > \int_2^3 g(x) dx$. Is this statement correct? Explain your answer.



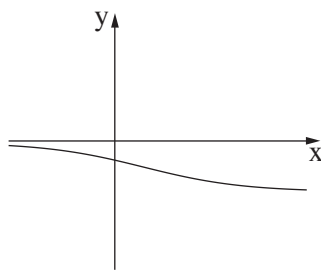
II



I



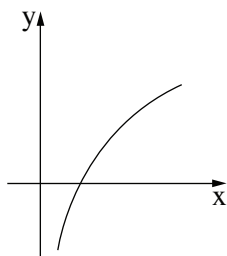
IV



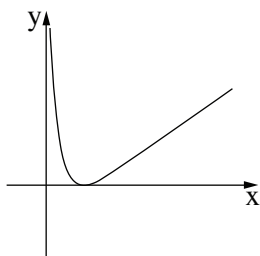
III

5. Consider the function $f(x) = \ln(x^n)$ where n is an even natural number.

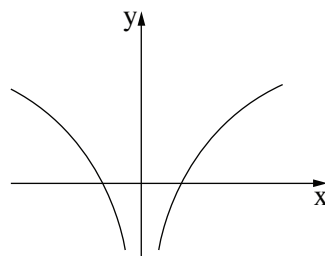
- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
(2) Below are four graphs, I–IV . Determine which one represents function $f(x)$.
Explain your answer.



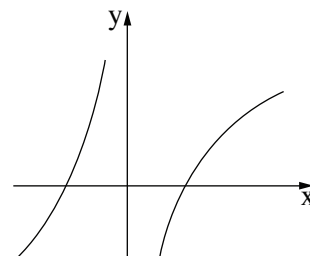
IV



III



II



I

Substitute $n = 2$ in function $f(x)$, and answer Items 6–7.

Consider the function $g(x) = (f(x))^2 - 4$, defined for all $x \neq 0$.

- ב. (1) Find the coordinates of the intersection points of the graph of function $g(x)$ with the x -axis.
(2) Find the coordinates of the extrema [נקודות קיצון] of function $g(x)$, and determine what type of extrema they are.
ג. Sketch a graph of function $g(x)$.

Consider the function $k(x) = \frac{g'(x)}{g(x)}$, on the interval $x > 0$.

- ד. (1) Find the domain of function $k(x)$.
(2) Calculate the area bounded by the graph of function $k(x)$, the x -axis, and the lines $x = e^2$ and $x = e^3$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך