The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2025
Exam Number: 035571
Appendix: Formula sheets for 5 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"ה, 2025
מספר השאלון: 035571
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

תוכנית חדשה

Note: This exam has special instructions. Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

Mathematics

מתמטיקה

5 Points – 1st Exam

5 יחידות לימוד – שאלון ראשון

Instructions

א. Duration of the exam: Four and a quarter hours.

ב. Exam structure and breakdown of points:
This exam has four parts and a total of eight questions.

Part One:	Short questions
Part Two:	Induction, Sequences, and Probability
Part Three:	Planar Geometry and Planar Trigonometry
Part Four:	Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions

Answer five questions, one question at least from Part One or Part Two, and at least one question from each of Part Three and Part Four.
$5 \times 20 = 100$ points.

ג. Material that may be used during the exam:
(1) A non-graphing calculator.
Do not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or may lead to the disqualification of your exam.

הוראות

א. משך הבחינה: ארבע שעות ורבע.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה ארבעה פרקים, ובהם שמונה שאלות.
פרק ראשון — שאלות קצרות
פרק שני — אינדוקצייה, סדרות והסתברות
פרק שלישי — גאומטרייה וטריגונומטרייה במישור
פרק רביעי — חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות שורש, של פונקציות רציונליות ושל פונקציות טריגונומטריות
יש לענות על חמש שאלות, על שאלה אחת לפחות מן הפרק הראשון או השני ועל שאלה אחת לפחות מכל אחד מן הפרקים השלישי והרביעי.
$5 \times 20 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer five of the questions 1–8, at least one question from Part One or Part Two, and at least one question from each of Part Three and Part Four (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Short Questions

1. Answer two of the four items א–ד below. If you answer more than two items, only the first two answers in your answer booklet will be graded.

א. **(1)** Prove by mathematical induction, or by any other method, that the following expression is satisfied for all natural n :

$$2^2 + 4^2 + 6^2 + 8^2 + ... + (2n)^2 = \frac{2n(2n+1)(n+1)}{3}$$

(2) Calculate the value of the sum: $4^2 + 6^2 + 8^2 + ... + 100^2$.

ב. Consider the trapezoid $ABCD$ $(AB \parallel CD)$ in the figure on the right.

Point O is the intersection point of the trapezoid's diagonals.

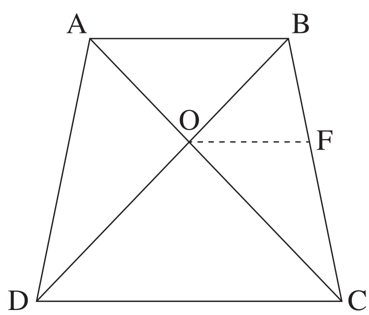
We know that $\sphericalangle BAO = \sphericalangle CDO$.

(1) Prove that Trapezoid $ABCD$ is an isosceles trapezoid.

Point F is located on Side BC such that Segment OF is parallel to the trapezoid's bases.

We know that the trapezoid's diagonals are perpendicular to each other, and that $DO = 4$, $BO = 3$.

(2) Find the length of Segment OF .



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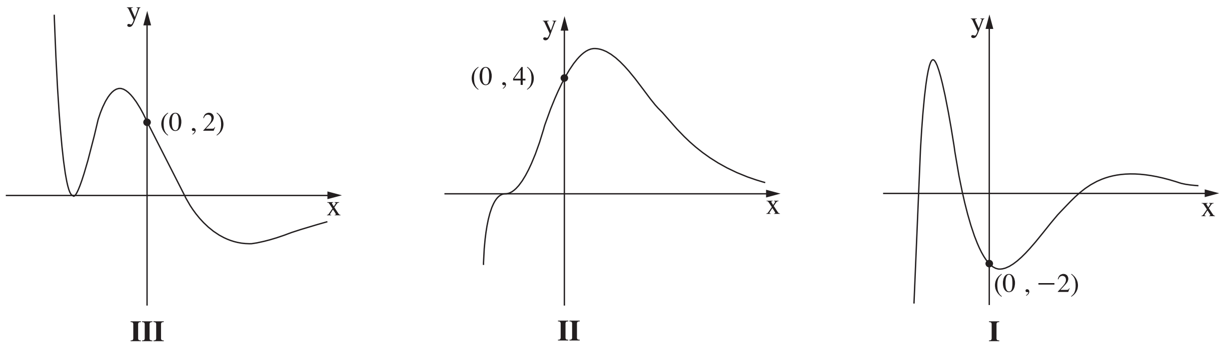
ג. Function $f(x)$, its derivative function $f'(x)$, and its second derivative $f''(x)$ are defined for all x.

Consider the three graphs I–III. One of them describes function $f(x)$, one describes the derivative function $f'(x)$, and one describes the second derivative $f''(x)$.

Each of the graphs shows all the extrema and all intersection points with the x-axis.

Each graph shows the coordinates of its intersection point with the y-axis.



(1) Match each function ($f(x)$, $f'(x)$, and $f''(x)$) with the graph that describes it. Explain your answer.

(2) How many inflection points [נקודות פיתול] does function $f(x)$ have? Explain your answer.

The coordinates of the maximum of function $f(x)$ are $(a, 5)$.

(3) Find the area bounded by the graph of function $f'(x)$, the graph of function $f''(x)$, the line $x = a$, and the y-axis.

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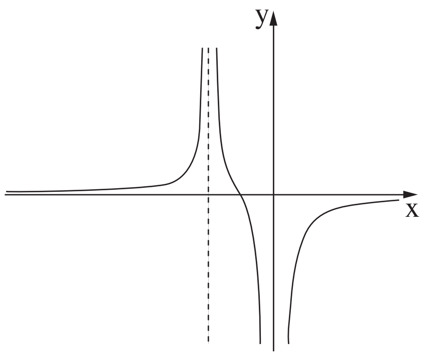
ד. Consider the function $f(x)=\frac{2(x+1)}{(x^2+2x)^2}$ defined on the domain $x \neq 0$, $x \neq -2$.

(1) Find the positive intervals of function $f(x)$.

(2) Determine which of the graphs I–IV below describes function $f(x)$. Explain your answer.

(3) Calculate the area bounded by the graph of function $f(x)$, the x-axis, and the lines $x=1$ and $x=4$.



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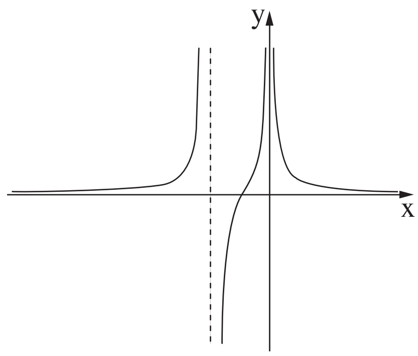

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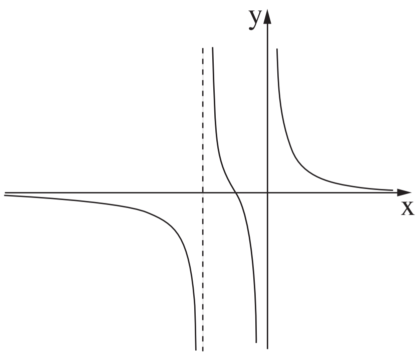

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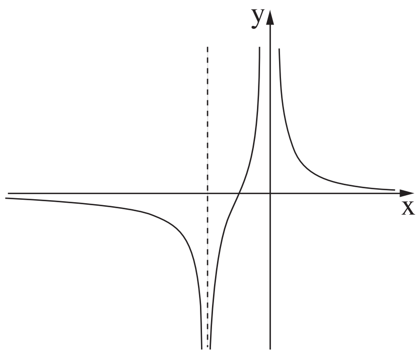

III

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Part Two – Induction, Sequences, and Probability

2. Consider an infinite geometric sequence A of terms $a_1, a_2, a_3, \dots$.

We know that $2a_2 + 8 = a_4$; $\frac{a_4}{a_2} = 4$.

א. Find the value of a_3 (find both possibilities).

We know that Sequence A is neither increasing nor decreasing.

We use the terms of Sequence A to build a new infinite sequence, B .

We know that the terms of Sequence B satisfy $b_n = \frac{1}{a_n \cdot a_{n+1}}$ for all natural n .

ב. Prove that Sequence B is a geometric sequence, and find its common ratio.

We use the terms of Sequence A to build another infinite geometric sequence, C .

The terms of Sequence C are $\frac{k}{a_1 \cdot a_2}, \frac{k}{a_3 \cdot a_4}, \frac{k}{a_5 \cdot a_6}, \dots$, $k \neq 0$ is a parameter.

ג. **(1)** Find the common ratio of Sequence C .

(2) Find for which values of k Sequence C is increasing. Explain your answer.

Let S_B be the sum of Sequence B , and S_C the sum of Sequence C .

We know that $S_C = 4 \cdot S_B$.

ד. Find the value of k .

3. Elections were held in a large country. Voters in these elections could vote either for Party A or for Party B.

Let P be the probability that a random voter voted for Party A $(0 < P < 1)$.

We randomly select 3 voters.

We know that the probability that exactly one of them voted for Party A is 2 times higher than the probability that all three voted for Party B.

א. Find the value of P .

We randomly select 4 voters.

ב. We know that all four voted for the same party. What is the probability that they voted for Party A?

Some voters are older adults and the rest are young adults.

We know that 40% of the older voters voted for Party B and 15% of the young voters voted for Party A.

ג. What is the probability of randomly selecting a young adult from among all the voters?

After the elections, a phone survey was conducted among the voters, in which Danny, one of the interviewers, called only young adults. He randomly called one voter after another, and stopped making calls immediately after he had interviewed one young adult who had voted for one party and another young adult who had voted for the other party.

ד. What is the probability that Danny called exactly 5 young adults?

/continued on page 6/

Part Three – Planar Geometry and Planar Trigonometry

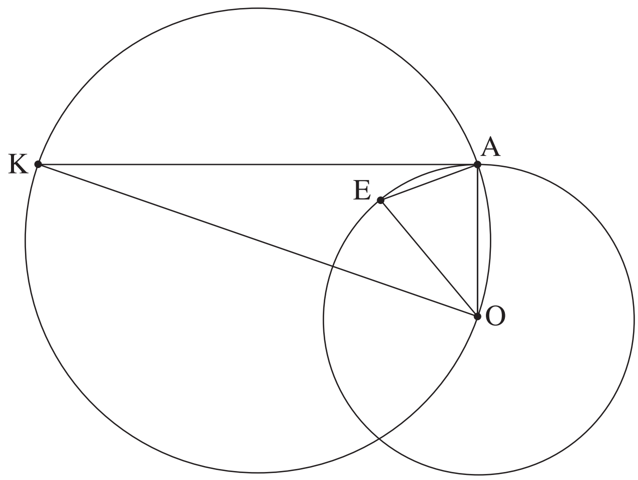
4. Consider a large circle of radius R and a small circle of center O and radius r, as shown in the figure.

Point O is located on the large circle.

Point A is one of the intersection points of the two circles, as shown in the figure.

A tangent to the small circle is drawn through Point A. The tangent intersects the large circle at Point K.

Point E is located on the small circle inside Triangle KAO.



א. Prove that $\sphericalangle AOE = 2\sphericalangle KAE$.

The extension of Segment AE intersects Segment OK at Point M.
We know that Point M is the midpoint of Segment OK.

ב. Prove that Point M is the center of the large circle.

ג. Prove that $\Delta MOA \sim \Delta OEA$.

We know that $R = 1.5r$.
Let S be the area of Triangle MEO.

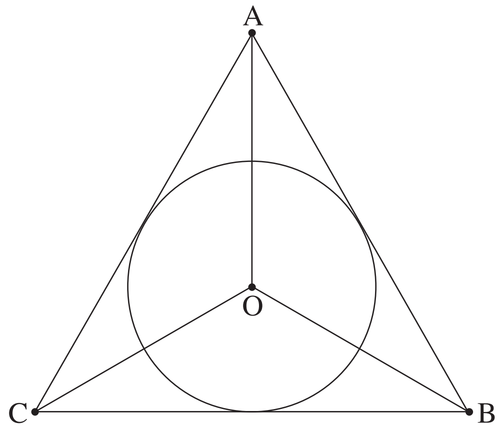
ד. Express the area of Triangle OKA in terms of S.

5. A circle of center O and radius r is inscribed in an isosceles triangle [משולש שווה שוקיים] ABC $(AC = AB)$ (see figure).

Let $\sphericalangle ACB = 2\alpha$.

א. **(1)** Express the length of Segment CO in terms of r and α.

(2) Express the length of Side AC in terms of r and α.



We know that the length of Side AC is $\sqrt{3}$ times greater than the length of Segment CO.

ב. Find the value of α.

Substitute $\alpha = 30^{o}$ and answer Items ג–ד.
The circle intersects Segment BO at Point K.
We know that the length of Segment CK is $\sqrt{175}$.

ג. Find the value of r.

Point E is located on Side CB.
We know that the area of Triangle CKE is 18.

ד. Calculate the length of Segment BE.

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Part Four – Differential and Integral Calculus of Polynomials, Root Functions, Rational Functions, and Trigonometric Functions

6. Consider the function $f(x)=\dfrac{x}{\sqrt{x^2-a^2}}$; a is a positive parameter.

Answer Items א–ו. Express your answers in terms of a , if necessary.

א. **(1)** Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

ב. Prove that function $f(x)$ is odd.

ג. Find the intervals on which function $f(x)$ is increasing and decreasing (if there are any).

ד. Sketch a graph of function $f(x)$.

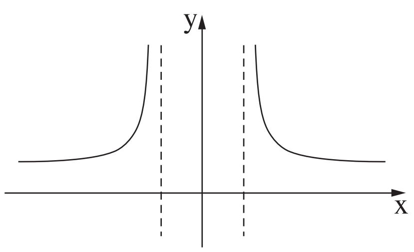
Consider the function $g(x)=\dfrac{1}{(f(x))^2}$. The domain of function $g(x)$ is the same as the domain of function $f(x)$.

ה. **(1)** Find the intervals on which function $g(x)$ is increasing and decreasing.

(2) Determine which of the graphs I–IV at the end of this question describes function $g(x)$. Explain your answer.

We know that the area bounded by the graph of function $g(x)$, the x-axis, and the lines $x = 2a$ and $x = 3a$ is 7.5 .

ו. Find the value of a .



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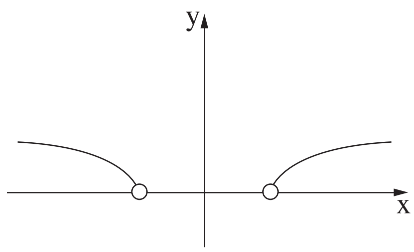

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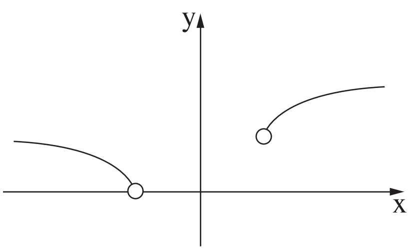

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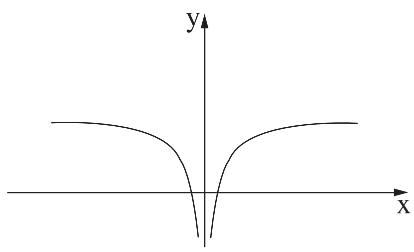

III

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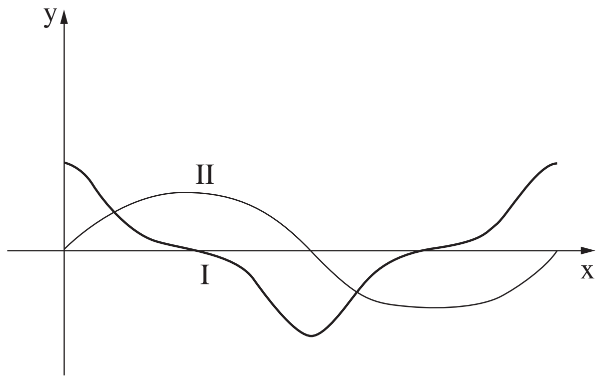
7. The figure on the right describes two graphs, I–II , on the interval $0 \le x \le 2\pi$.

One of the graphs describes function $f(x)$, and the other describes its derivative function $f'(x)$. The functions $f(x)$ and $f'(x)$ are defined for all x on the given interval.



א. Which of the graphs, I or II , describes function $f(x)$? Explain your answer.

We know that $f(x) = \frac{\sin x}{1 + (\sin x)^2}$ is defined on the domain $0 \le x \le 2\pi$.

ב. **(1)** Find the coordinates of the intersection points of the graph of function $f(x)$ with the x-axis.

(2) Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.

Consider the function $g(x) = \left| f(x) - 0.4 \right|$, defined on the domain $0 \le x \le 2\pi$.

ג. Find the coordinates of the intersection points of the graph of function $g(x)$ with the x-axis.

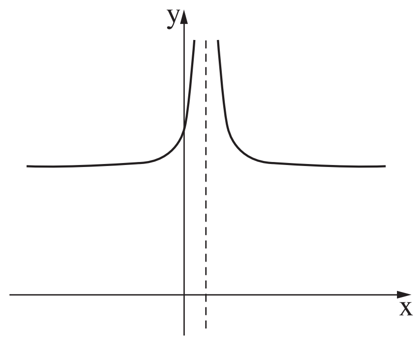
ד. **(1)** Sketch a graph of function $g(x)$.

(2) Find the coordinates of all the extrema of function $g(x)$, and determine what type of extrema they are.

8. Consider the graph of function $f(x) = \frac{a}{(x-1)^2} + 9$, shown in the figure below.

Function $f(x)$ is defined on the domain $x \ne 1$. a is a positive parameter.

א. Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.



Point C is located on the graph of function $f(x)$, and its x-coordinate is 2 .

A tangent to the graph of function $f(x)$ was drawn through Point C .

ב. Express the equation of the tangent in terms of a .

The tangent intersects the x-axis at Point A and the line $x = 1$ at Point B .

Consider Point D whose coordinates are $(1, 0)$.

ג. Express the area of Triangle ADB in terms of a .

ד. Find the value of a for which the area of Triangle ADB is smallest (minimum).

Good Luck!

בהצלחה!