

Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד – שאלון ראשון

Instructions

א. Duration of the exam: Four hours.

ב. Exam structure and breakdown of points:

This exam consists of three parts with a total of eight questions:

- Part One – Algebra, Analytical Geometry, and Probability
Part Two – Planar Geometry and Planar Trigonometry
Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer **five** questions, with at least **one** question from each part – $5 \times 20 = 100$ points.

הוראות

א. משך הבחינה: ארבע שעות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

- פרק ראשון – אלגברה, גאומטריה אנליטית והסתברות
פרק שני – גאומטריה וטריגונומטריה במישור
פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש
יש לענות על **חמש** שאלות, על שאלה **אחת** לפחות מכל פרק
— $100 = 20 \times 5$ נקודות.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. Do not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי / לועזי–עברי.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain **all** of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את **כל** הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write **only in the answer booklet**. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

יש לכתוב **במחברת הבחינה בלבד**. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer five of the questions 1–8, with at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Algebra, Analytical Geometry, and Probability

1. Dafna and Yael set off at the same time to walk along the same trail. The trail is 13.5 km long. Dafna walked at a constant speed without stopping until the end of the trail. Yael walked at the same speed as Dafna for one hour, and then stopped for a half-hour break. After this break, Yael continued walking until the end of the trail at a speed that was 0.5 km/h faster than her initial speed. She reached the end of the trail 18 minutes after Dafna.
 - a. Find Dafna's walking speed.
 - b. At what speed should Yael have walked after her break in order to reach the end of the trail at the same time as Dafna?

2. The figure on the right shows a circle whose center M is located on the x -axis.

The line $y = \frac{3}{4}x + \frac{7}{4}$ is tangent to the circle at Point D .

We know that the y -coordinate of Point D is 4.

- a. Find the equation of Line MD .
- b. Find the equation of the circle.

Point A is the right-hand point of intersection of the circle with the x -axis, as shown in the figure.

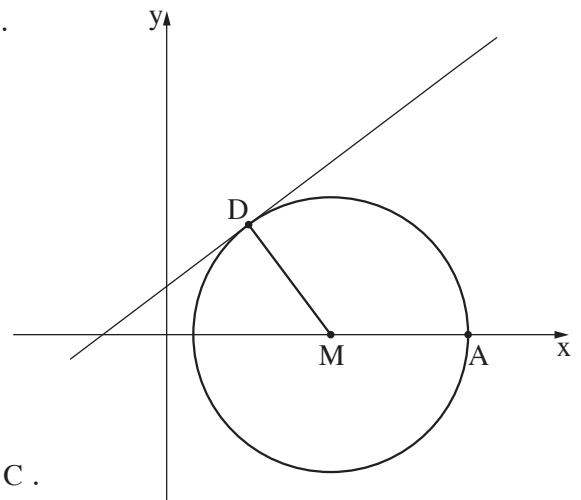
A tangent to the circle is drawn through Point A .

The tangent to the circle through Point A and the tangent to the circle through Point D intersect at Point C .

- a. (1) Find the coordinates of Point C .
- (2) Find the perimeter of Quadrilateral $DMAC$.

Quadrilateral $DMAC$ is inscribed in a circle.

7. Is the radius of the circle that circumscribes Quadrilateral $DMAC$ greater than the radius of the circle whose center is M ? Explain your answer.



3. Some of the 11th grade students in a certain school are boys and the rest are girls. Some of the 11th grade students volunteer in their spare time and the rest do not.

75% of the 11th grade students volunteer.

$\frac{3}{5}$ of the volunteers are girls.

We know that the number of girls who volunteer is 3 times greater than the number of girls who do not volunteer.

- א. What is the probability of randomly selecting a girl who volunteers from among the 11th grade students?
- ב. Find the percentage of boys in the 11th grade.

A student is randomly selected from among the 11th grade students.

- ג. We know that the selected student is a boy. What is the probability that he volunteers?

Dana wrote an article for the 11th grade paper in which she stated: "The percentage of volunteers among the 11th grade girls is greater than the percentage of volunteers among the 11th grade boys."

- ד. Is Dana's statement correct? Explain your answer.

Part Two – Planar Geometry and Planar Trigonometry

4. Quadrilateral ABCD is inscribed in a circle.

Point F is located on the extension of Side AD such that CF is tangent to the circle.

We know that Chord AC bisects Angle BAD (see figure).

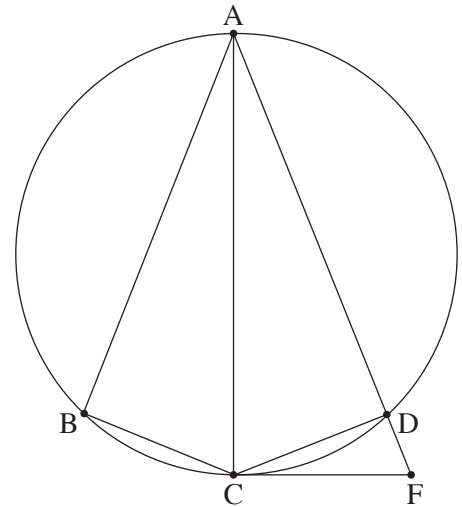
- א. Prove that $\angle ABC = \angle CDF$.
ב. (1) Prove that $\triangle ABC \sim \triangle CDF$.
(2) Prove that $(BC)^2 = AB \cdot DF$.

We know that the area of Triangle ABC is 6.25 times greater than the area of Triangle CDF and that $DF = 4$.

- ג. Find the lengths of Segments BC and AB .

We know that $\angle CDF = 90^\circ$.

- ד. Find the length of the radius of the circle.



5. The figure on the right shows Triangle ABC .

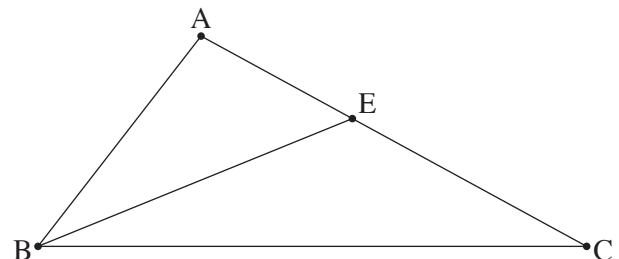
Point E is located on Side AC .

We know that $AB = 12$, $AC = 20$, $BC = 25$.

- א. Find the size of Angle BAC .

We know that the length of Segment AE is equal to the radius of the circle that circumscribes Triangle ABE .

- ב. Find the size of Angle ABE .
ג. Find the length of Segment EC .

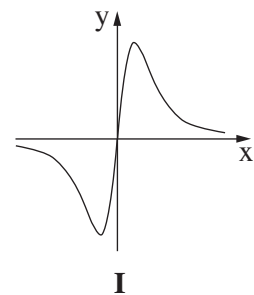
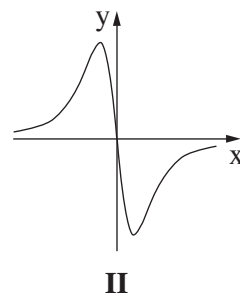
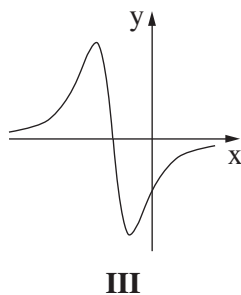
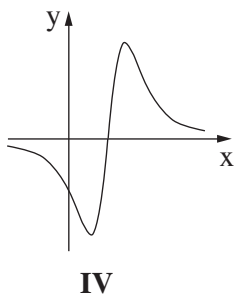


Point K is located on Side BC such that the area of Triangle ECK is 32 .

- ד. Find the length of Segment CK .

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

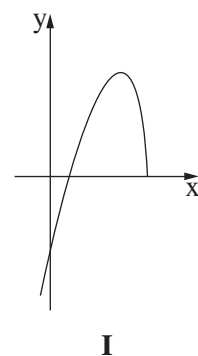
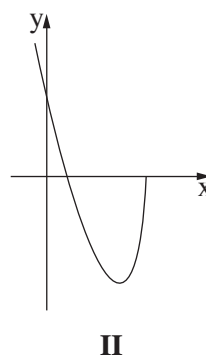
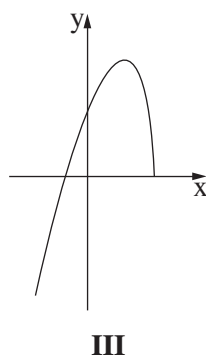
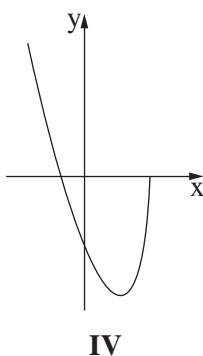
6. Consider the function $f(x) = \frac{x^2 - 9}{x^2 + 1} + 1$ defined for all x .
- א. Find the equation of the asymptote of function $f(x)$ that is parallel to the x -axis.
 - ב. (1) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
(2) List the intervals on which function $f(x)$ is increasing and decreasing.
 - ג. Find the coordinates of the intersection points of the graph of function $f(x)$ with the x -axis.
 - ד. Sketch a graph of function $f(x)$.
 - ה. (1) Determine which of the figures I–IV below describes the graph of the derivative function $f'(x)$. Explain your answer.
(2) Calculate the area bounded by the graph of the derivative function $f'(x)$, the x -axis, and the lines $x = 2$ and $x = -2$.



7. Consider the function $f(x) = (3x - 9) \cdot \sqrt{15 - x}$.
- א. Find the domain [תחום ההגדרה] of function $f(x)$.
 - ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
 - ג. Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
 - ד. Sketch a graph of function $f(x)$.

Consider the function $g(x) = f(x + 6)$, defined on the domain $x \leq 9$.

- ה. (1) Which of the graphs I–IV below describes function $g(x)$? Explain your answer.
(2) What are the coordinates of the local extremum [נקודת קיצון פנימית] of function $g(x)$?



8. The figure on the right describes the function

$$f(x) = \frac{a}{x-2}, \text{ defined on the domain } x \neq 2.$$

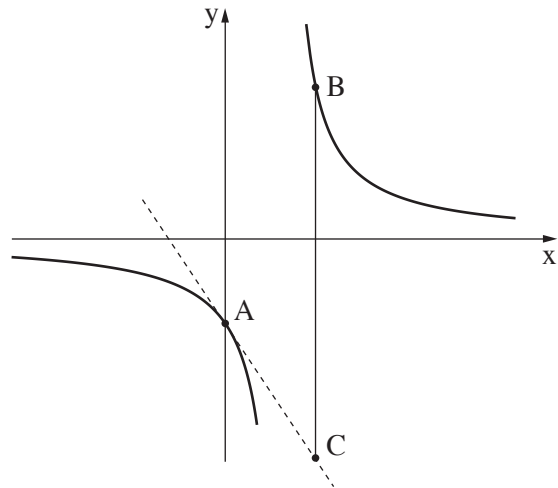
a is a non-zero parameter.

Point A is the intersection point of the graph of function $f(x)$ with the y -axis.

A tangent to the graph of function $f(x)$ is drawn through Point A .

We know that the slope of the tangent is -1.5 .

א. Find the value of a .



Substitute $a = 6$ in function $f(x)$, and answer Items ב-ג.

ב. Find the equation of the tangent.

Point B is located on the graph of function $f(x)$, on the interval $x > 2$.

Point C is located on the tangent such that Segment BC is parallel to the y -axis.

Let t be the x -coordinate of Point B .

ג. (1) Express the length of Segment BC in terms of t .

(2) Find the value of t for which the length of Segment BC is smallest (minimum).

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך