

תוכנית חדשה

Note: This exam has special instructions. Answer the questions according to these instructions.

שים לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions

הוראות

א. Duration of the exam: Two and a quarter hours.

א. משך הבחינה: שתיים ורבע.

ב. Exam structure and breakdown of points:

ב. מבנה השאלון ומפתח ההערכה:

This exam has two parts, with a total of five questions.

בשאלון זה שני פרקים, ובהם חמש שאלות.

Part One – Sequences, Spatial Geometry, and Growth and Decay

פרק ראשון – סדרות, גאומטריה במרחב וגדילה ודעיכה

Part Two – Differential and Integral Calculus of Exponential and Logarithmic Functions

פרק שני – חשבון דיפרנציאלי ואינטגרלי של פונקציות מעריכיות ולוגריתמיות

Answer three questions, at least one from each part
– $3 \times 33\frac{1}{3} = 100$ points.

יש לענות על שלוש שאלות, על שאלה אחת לפחות מכל פרק
– $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) Formula sheets (attached).

(2) דפי נוסחאות (מצורפים).

(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

(3) מילון עברי-לועזי / לועזי-עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each question on a new page.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.

If you use any draft paper outside the answer booklet, your exam may be disqualified.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Sequences, Spatial Geometry, and Growth and Decay

1. Consider an arithmetic sequence A of 57 terms: $a_1, a_2, a_3, \dots$.

We know that $a_2 + a_4 = a_6$.

The last term in Sequence A is 171.

- א. Find the sequence's common difference and a_1 .

Consider an infinite geometric sequence B whose terms are: $b_1, b_2, b_3, \dots$.

We know that $b_1 = a_6$, $3 \cdot b_3 = a_8$.

- א. Find the common ratio of Sequence B (find both possibilities).

We know that Sequence B neither increases nor decreases.

- א. Find the sum of the even-position terms in Sequence B .

2. The figure to the right shows Pyramid $ABCE$.

Point D is the midpoint of Edge BC .

Point M is located on ED such that $\overrightarrow{EM} = \frac{2}{3}\overrightarrow{ED}$.

Edge AE is a height of the pyramid.

We know that $\angle BAC = 90^\circ$.

Let $\overrightarrow{AB} = \underline{u}$, $\overrightarrow{AC} = \underline{v}$, $\overrightarrow{AE} = \underline{w}$.

- א. Express the vectors $\overrightarrow{BC}$, $\overrightarrow{ED}$, and $\overrightarrow{AM}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.

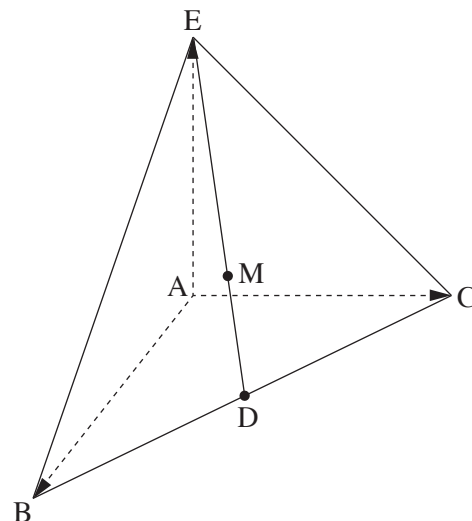
We know that $|\underline{w}| = |\underline{v}| = |\underline{u}| = 12$.

- א. (1) Prove that $\overrightarrow{AM}$ is perpendicular to $\overrightarrow{ED}$.
(2) Prove that $\overrightarrow{AM}$ is perpendicular to Plane BEC .

We know: $A(0, 0, 0)$.

Vertex B is located on the positive part of the x -axis, Vertex C is located on the positive part of the y -axis, and Vertex E is located on the positive part of the z -axis.

- א. (1) What are the coordinates of Points B , C , and E ?
(2) Find the coordinates of Points D and M .
א. Calculate the area of Triangle AMD .



3. Gardeners in two nurseries – Nursery A and Nursery B – started potting plants on the same day. On that day, the gardeners in Nursery A potted 600 plants and the gardeners in Nursery B potted a plants.

The number of plants in Nursery A increased by 5% every week, and the number of plants in Nursery B increased by 10% every week.

After a while the number of plants in Nursery A equalled the number of plants in Nursery B.

- א. Below are two statements, I–II. Determine which statement is correct. Explain your answer.

I $a < 600$

II $a > 600$

17 weeks from when the gardeners started potting, the number of plants in Nursery B was 2 times greater than the number of plants in Nursery A.

- ב. Find the value of a .

17 weeks from when the gardeners started potting, both nurseries stopped potting, and started selling their plants.

From the day they started selling, the number of plants in Nursery B decreased by a fixed percentage every week.

5 weeks after they started selling, the number of plants in Nursery B was 53% of the number on the day they started selling.

- ג. Find the percentage by which the number of plants in Nursery B decreased every week.

From the day they started selling, the number of plants in Nursery A decreased by 8% every week.

- ד. Find how much time passed from the day they started selling until the number of plants in Nursery A equalled the number of plants in Nursery B.

Part Two – Differential and Integral Calculus of Exponential and Logarithmic Functions

4. Consider function $f(x) = \frac{e^{2x}}{x^2 - 4x + a}$; a is a parameter.

Function $f(x)$ has a vertical asymptote whose equation is $x = 2$.

א. Find the value of a .

Substitute $a = 4$ in function $f(x)$, and answer Items ב–ד.

- ב. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
(2) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).
(3) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.
ג. Sketch a graph of function $f(x)$.

Consider function $h(x) = -f(x) + e^8$ whose domain is the same as that of function $f(x)$.

- ד. How many intersection points does the graph of function $h(x)$ have with the x -axis?
Explain your answer.

5. The function $f(x)$ and its derivative function $f'(x)$ are defined on the domain $x > 0$.

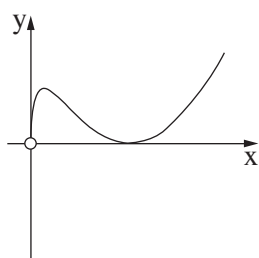
Function $f(x)$ has only one minimum and only one maximum.

We know that the minimum is located to the right of the maximum.

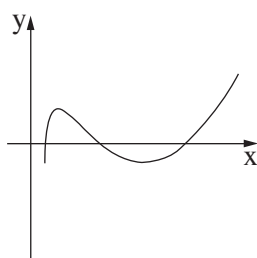
- א. One of the graphs I–IV at the end of this question describes the derivative function $f'(x)$.
Determine which one it is, and explain your answer.

We know that $f(x) = 3x \cdot (\ln x)^2$.

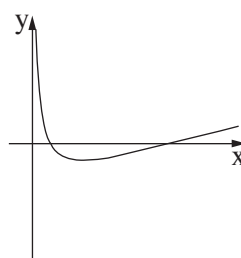
- ב. (1) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).
(2) Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
ג. Determine which of the graphs I–IV at the end of this question describes function $f(x)$.
ד. Find the area bounded by the graph of the derivative function $f'(x)$ and the x -axis.



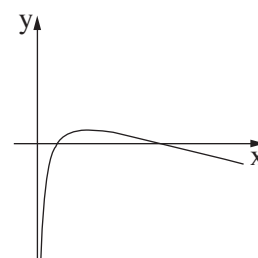
IV



III



II



I

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך