



Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד - שאלון ראשון

Instructions

הוראות

א. Duration of the exam: Four hours.

א. משך הבחינה: ארבע שעות.

ב. Exam structure and breakdown of points:

This exam consists of three parts, with a total of eight questions.

Part One – Statistics and Probability

Part Two – Geometry

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer five questions, at least one question from each part – $5 \times 20 = 100$ points.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

פרק ראשון – סטטיסטיקה והסתברות

פרק שני – גאומטריה

פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים,

של פונקציות רציונליות ושל פונקציות שורש

יש לענות על חמש שאלות, על שאלה אחת לפחות מכל פרק

– $20 \times 5 = 100$ נקודות.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות

במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות

במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי / לועזי–עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת

את שלבי הפתרון, גם כאשר החישובים מתבצעים

בעזרת מחשבון.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט

Lack of detail may lower your score or lead to the disqualification of your exam.

ובצורה ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת

הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer five of the questions 1–8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Statistics and Probability

1. The results of a long jump competition in a certain school were normally distributed.

The average long jump distance in this competition was 4.6 meters.

69.2% of the competitors jumped a distance less than 4.8 meters.

- א. Find the standard deviation of jump distance.

Competitors who jumped further than 5.24 meters won a medal.

- ב. What percentage of the competitors won a medal?

We know that 11 competitors won a medal.

- ג. According to the standard normal distribution table, how many competitors participated in the long jump competition?

The same school also ran a high jump competition. The results in this competition were normally distributed, too. The average jump height was 1.5 meters and the standard deviation was 0.2 meters.

Alon competed in both competitions. His long jump result was 5 meters and his high jump result was 1.75 meters.

- ד. In which competition, the long jump or the high jump, did Alon's result rank higher in comparison to the other competitors in that competition? Explain your answer.

2. A real-estate agent wanted to check the linear relationship between the size of apartments sold last month in a certain neighborhood (x variable) and the apartments' selling prices (y variable). The following table shows data for 5 apartments that the real-estate agent checked.

Apartment size (m^2) – x variable	Apartment price (millions of shekels) – y variable
80	1
88	1.8
96	2.7
110	3.4
116	4.6

- א. (1) Find the average apartment size.
(2) Find the standard deviation of apartment size.

The agent calculated the average and standard deviation of the y variable and found that $\bar{y} = 2.7$ and $S_y = 1.249$.

- א. Find the correlation coefficient r between the two variables.
ב. Find the equation of the regression line for predicting y according to x .
ג. Based on the regression line, what is the predicted price of an apartment whose size is $100 m^2$?

Following the rise in prices of goods and services, the price of each apartment increased by 0.2 million shekels.

- ה. Determine whether each of the measures (1) and (2) below, increased, decreased, or remained the same. Explain each of your answers.
(1) standard deviation of the y variable.
(2) correlation coefficient r .

3. Some of the 11th grade students in a certain school are boys and the rest are girls. Some of the 11th grade students volunteer in their spare time and the rest do not.

75% of the 11th grade students volunteer.

$\frac{3}{5}$ of the volunteers are girls.

We know that the number of girls who volunteer is 3 times greater than the number of girls who do not volunteer.

- א. What is the probability of randomly selecting a girl who volunteers from among the 11th grade students?
ב. Find the percentage of boys in the 11th grade.

A student is randomly selected from among the 11th grade students.

- ג. We know that the selected student is a boy. What is the probability that he volunteers?

Dana wrote an article for the 11th grade paper in which she stated: "The percentage of volunteers among the 11th grade girls is greater than the percentage of volunteers among the 11th grade boys."

- ד. Is Dana's statement correct? Explain your answer.

Part Two – Geometry

4. The figure on the right describes a right triangle ABC
($\angle ACB = 90^\circ$).

Vertex B is located on the y -axis.

We know that the coordinates of Vertex A are $(-16, 0)$
and the equation of Side CB is $y = -\frac{2}{5}x + 11$.

- א. Find the equation of Side AC .
- ב. Find the coordinates of Vertex C .

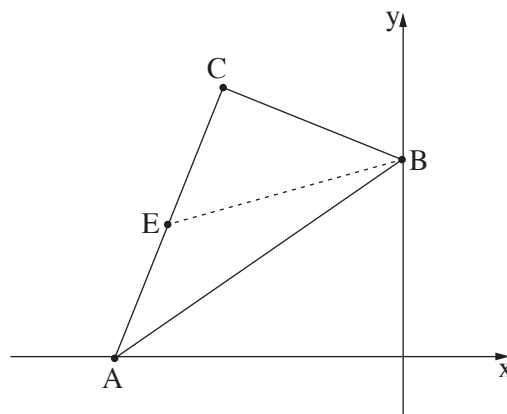
BE is the median to Side AC .

- ג. (1) Find the length of Segment EC .
- (2) Find the size of Angle CEB .

Point K is located on Segment EB .

We know that the area of Triangle AEK is 18.

- ד. Find the length of Segment EK .



5. The figure on the right describes a circle of center M .
The circle intersects the positive side of the x -axis at Point A .

Point B is located on the circle such that AB is a diameter of the circle.

A tangent to the circle is drawn through Point B ,
intersecting the x -axis at Point C .

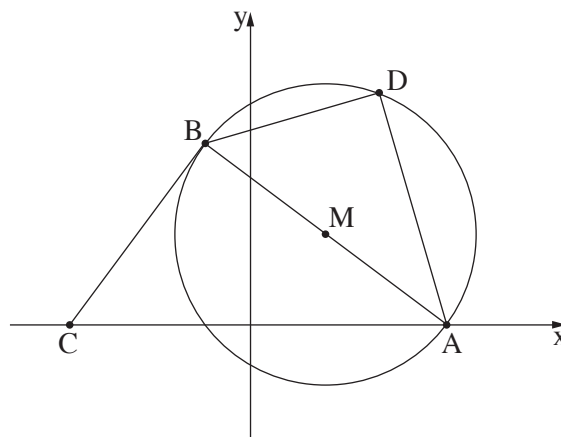
We know that the equation of the circle is

$$(x - 5)^2 + (y - 6)^2 = 100.$$

- א. Find the coordinates of Point A .
- ב. Find the equation of Tangent BC .
- ג. Find the length of Side AC .

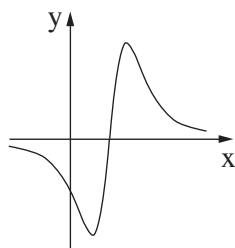
Point D is located on the circle such that AB bisects Angle DAC .

- ד. (1) Explain why $\angle ADB = 90^\circ$.
- (2) Prove that $\triangle ABC \sim \triangle ADB$.
- ה. By what factor is the area of Triangle ABC greater than the area of Triangle ADB ?

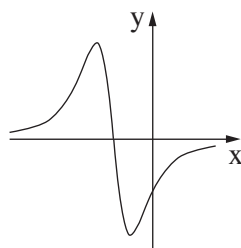


Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

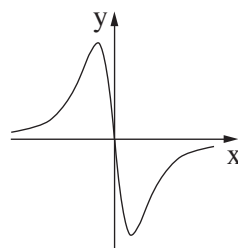
6. Consider the function $f(x) = \frac{x^2 - 9}{x^2 + 1} + 1$ defined for all x .
- א. Find the equation of the asymptote of function $f(x)$ that is parallel to the x -axis.
 - ב. (1) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.
(2) List the intervals on which function $f(x)$ is increasing and decreasing.
 - ג. Find the coordinates of the intersection points of the graph of function $f(x)$ with the x -axis.
 - ד. Sketch a graph of function $f(x)$.
 - ה. (1) Determine which of the figures I–IV below describes the graph of the derivative function $f'(x)$. Explain your answer.
(2) Calculate the area bounded by the graph of the derivative function $f'(x)$, the x -axis, and the lines $x = 2$ and $x = -2$.



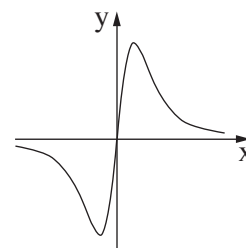
IV



III



II

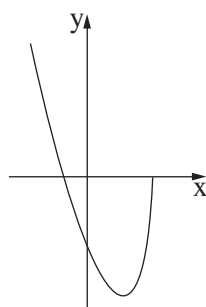


I

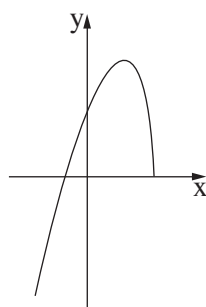
7. Consider the function $f(x) = (3x - 9) \cdot \sqrt{15 - x}$.
- א. Find the domain [תחום הגדרה] of function $f(x)$.
 - ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
 - ג. Find the coordinates of all the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
 - ד. Sketch a graph of function $f(x)$.

Consider the function $g(x) = f(x + 6)$, defined on the domain $x \leq 9$.

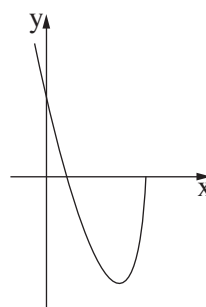
- ה. (1) Which of the graphs I–IV below describes function $g(x)$? Explain your answer.
(2) What are the coordinates of the local extremum [נקודת קיצון פנימית] of function $g(x)$?



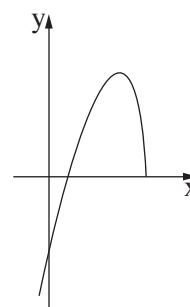
IV



III



II



I

8. The figure on the right describes the function

$$f(x) = \frac{a}{x-2}, \text{ defined on the domain } x \neq 2.$$

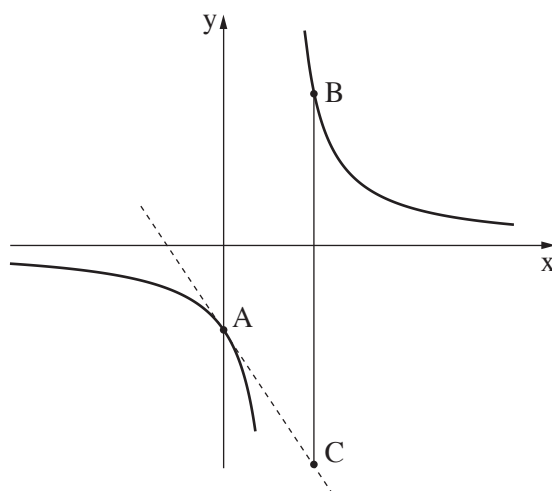
a is a non-zero parameter.

Point A is the intersection point of the graph of function $f(x)$ with the y-axis.

A tangent to the graph of function $f(x)$ is drawn through Point A.

We know that the slope of the tangent is -1.5 .

א. Find the value of a .



Substitute $a = 6$ in function $f(x)$, and answer Items ב-ג.

ב. Find the equation of the tangent.

Point B is located on the graph of function $f(x)$, on the interval $x > 2$.

Point C is located on the tangent such that Segment BC is parallel to the y-axis.

Let t be the x-coordinate of Point B.

- ג. (1) Express the length of Segment BC in terms of t .
- (2) Find the value of t for which the length of Segment BC is smallest (minimum).

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך