

Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

3rd Exam – 3 Points

Instructions

א. Duration of the exam: Two and a quarter hours.

ב. Exam structure and breakdown of points:

This exam has six questions on the topics – Algebra, Differential and Integral Calculus.

Answer four questions – each question is worth 27 points. In total – a maximum of 100 points.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to disqualification of your exam.

Write in the answer booklet only. Write "טיוטה" at the top of every page you use for draft work.

If you use pages outside the answer booklet for draft work, your exam may be disqualified.

מתמטיקה

3 יחידות לימוד – שאלון שלישי

הוראות

א. משך הבחינה: שתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים –

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

יש לענות על ארבע שאלות – לכל שאלה 27 נקודות.

סך הכול – 100 נקודות לכל היותר.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות

במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון

עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת

את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת

מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה

ברורה ומסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer four of the questions 1–6 (each question – 27 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. Michal, the school librarian, ordered mathematics and English textbooks from a certain bookstore.

The price of a mathematics textbook is 1.6 times higher than the price of an English textbook.

The price of one mathematics textbook and two English textbooks is 189 shekels in total.

- א. (1) Find the price of an English textbook.
(2) Find the price of a mathematics textbook.

Michal received a 15% discount on the price of a mathematics textbook (the price of the English textbook did not change).

Michal ordered 49 more mathematics textbooks than English textbooks.

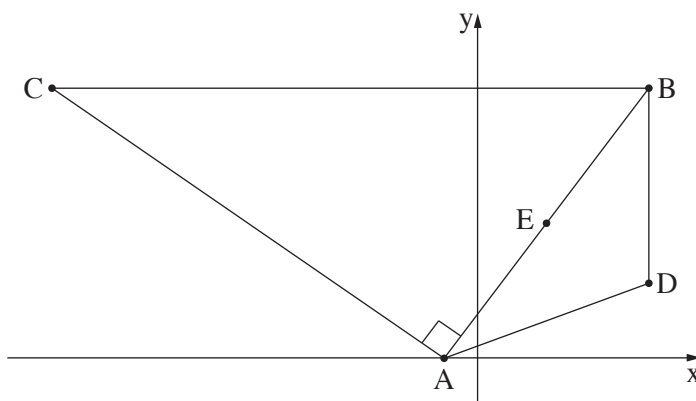
She paid a total of 9,198 shekels for all the mathematics and English textbooks she ordered.

- ב. (1) Find the price of a mathematics textbook after discount.
(2) Find how many English textbooks Michal ordered.

2. In the right triangle ABC ($\angle BAC = 90^\circ$),
Vertex A is located on the x -axis.
Point E is the midpoint of Side AB
(see figure).

We know $A(-2, 0)$, $E(4, 9)$.

- א. Find the coordinates of Vertex B .
ב. (1) Find the slope of Side AB .
(2) Find the equation of Side AC .



Side BC is parallel to the x -axis.

- ג. Find the coordinates of Vertex C .

We know $D(10, 5)$.

- ד. Show that Triangle ADB is an isosceles triangle [משולש שווה שוקיים].
ה. Calculate the perimeter of Quadrilateral $ADBC$.

3. Consider a circle of center M whose equation is $(x - 6)^2 + (y - 5)^2 = 90$.

א. What are the coordinates of the center of the circle, M ?

Point A is located on the circle in the first quadrant, as shown in the figure.

We know that the x -coordinate of Point A is 9.

ב. Find the y -coordinate of Point A .

A tangent to the circle was drawn through Point A .

The tangent intersects the y -axis at Point C .

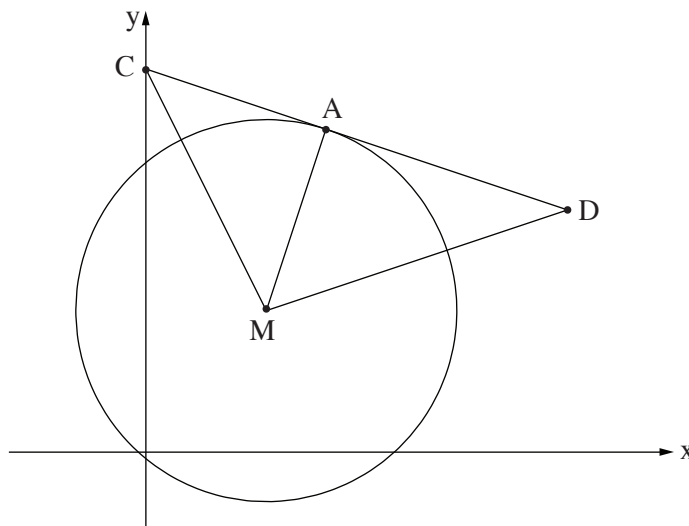
- ג. (1) Find the slope of AM .
- (2) Find the equation of the tangent.
- (3) Find the coordinates of Point C .

Point D is located on the tangent.

We know that the equation of the line MD is

$$y = \frac{1}{3}x + 3.$$

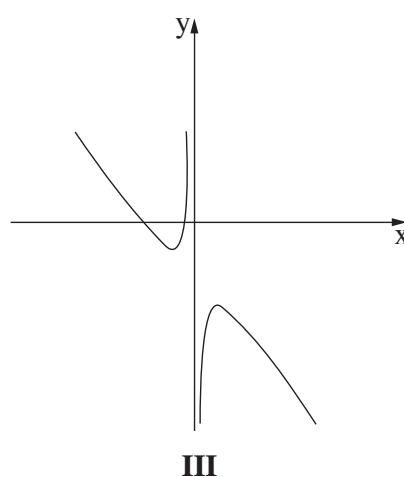
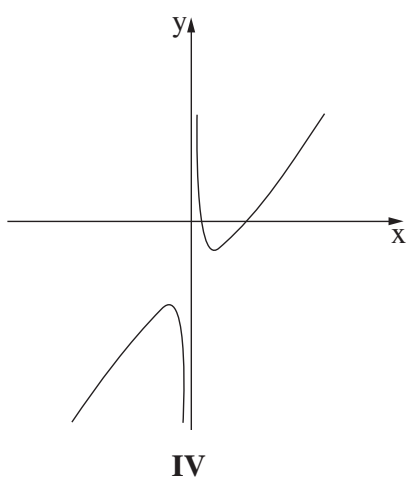
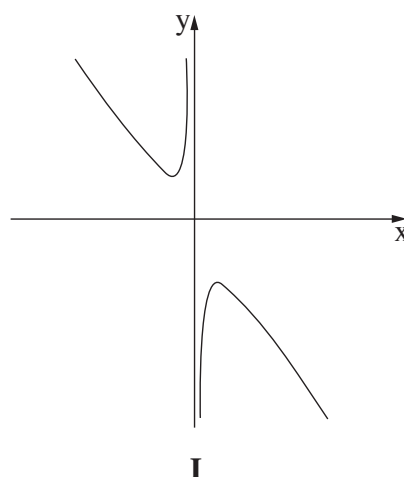
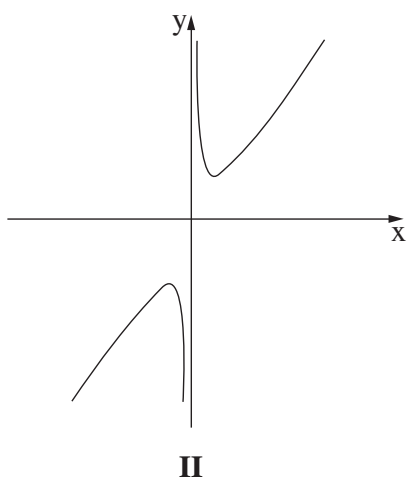
- ד. (1) Find the coordinates of Point D .
- (2) Find the area of Triangle CMD .



Differential and Integral Calculus

4. Consider the function $f(x) = 1.5x + \frac{6}{x} - 1$.

- א. What is the domain [תחום ההגדרה] of function $f(x)$?
- ב. Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
- ג. Find the intervals on which function $f(x)$ is increasing.
- ד. Determine which of the graphs I–IV at the end of the question describes function $f(x)$.
Explain your answer.
- ה. Is the slope of the tangent to the graph of function $f(x)$ at the point where $x = 2$ smaller than the slope of the tangent to this graph at the point where $x = -3$? Explain your answer.



5. Consider the function $f(x) = -x^2 + 12x - 27$.

Points A and B are the intersection points of the graph of function $f(x)$ with the x-axis, as shown in the figure.

Point C is the maximum of function $f(x)$.

א. Find the coordinates of Points A and B.

ב. Find the coordinates of Point C.

A line is drawn through Points B and C.

Consider two equations, I and II.

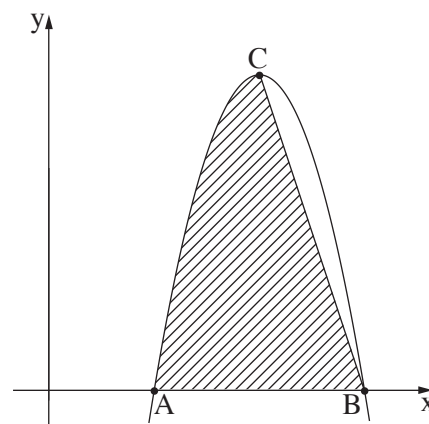
I $y = -4x + 36$

II $y = -3x + 27$

ג. Determine which of the equations, I or II, is the equation of Line BC. Explain your answer.

ד. Calculate the shaded area in the figure:

the area bounded by the graph of function $f(x)$, Line BC, and the x-axis.



6. Consider Square ABCD and Rectangle EFGH, shown in the figure to the right.

Let x be the side length of the square.

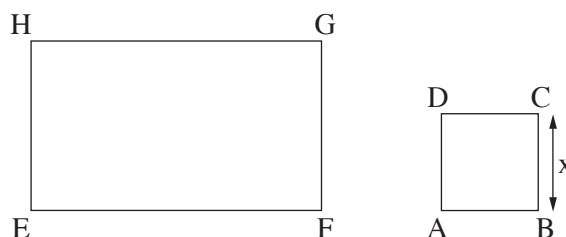
We know that the length of Side EF is 3 times greater than the square's side length and that the sum of the side lengths EF and GF is 12.

א. (1) Express the length of Side EF in terms of x .

(2) Express the length of Side GF in terms of x .

ב. Express the area of Rectangle EFGH in terms of x .

ג. Find the value of x for which the sum of the square and rectangle areas is a maximum.



Good Luck!

All rights reserved by the State of Israel.
Copying or publication without the consent of the
Ministry of Education is prohibited.

בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך