The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2025
Exam Number:35582
Appendix: Formula sheets for 5 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תשפ"ה, 2025
מספר השאלון: 35582
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

Note: This exam has special instructions. Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Two hours and forty-five minutes.

ב. Exam structure and breakdown of points:
This exam has two parts with a total of five questions.
Part One — Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers
Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

Answer three questions with at least one question from each part – $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות

א. משך הבחינה: שעתיים וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון — גאומטרייה אנליטית, וקטורים, טריגונומטרייה במרחב, מספרים מרוכבים
פרק שני — גדילה ודעיכה, פונקציות חזקה, פונקציות מעריכיות ולוגריתמיות
יש לענות על שלוש שאלות, לפחות על שאלה אחת מכל פרק
— $3 \times 33\frac{1}{3} = 100$ נקודות

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer three of the questions 1–5 with at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider the points $A(-9a\,,0)$ and $B(41a\,,0)$. a is a positive parameter.

Point P satisfies $\sphericalangle APB = 90^o$.

א. Show that the locus [המקום הגאומטרי] of all the points P obtained in this way is on a circle, and find the equation of the circle.

Express your answer in terms of a.

We know that the largest area of Triangle APB is 156.25.

ב. Find the value of a.

Substitute $a = \frac{1}{2}$ and answer Items ג–ד.

The midpoint of Segment AB is the focus of a parabola whose equation is in canonical form.

The parabola and the circle you found intersect in the first quadrant, at Point C.

ג. Find the coordinates of Point C.

The line ℓ is tangent to the parabola at Point C.

A tangent is drawn to the circle you found.

We know that this tangent is parallel to the line ℓ.

ד. Find the distance between the line ℓ and the tangent to the circle (find both possibilities).

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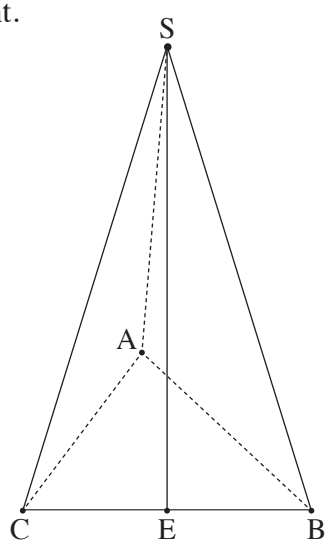
2. Consider the triangular pyramid SABC shown in the figure on the right.



Point E is the midpoint of Edge BC .

Let $\overrightarrow{EC} = \underline{v}$, $\overrightarrow{AE} = \underline{u}$.

We know that $|\underline{u} + \underline{v}| = |\underline{u} - \underline{v}|$.

א. Prove that $\overrightarrow{AE}$ is perpendicular to $\overrightarrow{BC}$.

Given: Segment SE is a height of the pyramid and its length is 16 ;

B (6 , 8 , 0) , A (0 , 0 , 0) .

Vertex C is located on the positive part of the x-axis, and the z-coordinate of Vertex S is positive.

ב. Find the coordinates of Vertices C and S .

Point F is located on Edge SC such that $\overrightarrow{SF} = \frac{1}{4}\overrightarrow{SC}$.

A line is drawn through Point F , parallel to Line AC and intersecting Plane ABS at Point M .

ג. **(1)** Find a vector form [הצגה פרמטרית] of Line FM .

(2) Find the coordinates of Point M .

ד. By what factor is the volume of Pyramid MABC greater than the volume of Pyramid FAEC ? Explain your answer.

3. Consider the complex number $w = r(\cos\theta + i \cdot \sin\theta)$, $r > 0$, $90^o < \theta < 180^o$.

א. Express a polar representation of the number $\frac{1}{w}$ in terms of r and θ .

Point A on the Gauss plane represents the number w , and Point B represents the number $\frac{1}{w}$.

Point O is the origin.

We know that the area of Triangle AOB is $\frac{\sqrt{2}}{4}$ and the angle AOB is an acute angle.

ב. Find the value of θ .

Consider two equations: I $z^8 = w^8$

II $z^8 = (w \cdot \overline{w})^4$

z is a complex number.

Consider a given convex polygon [מצולע קמור] whose vertices are all the points representing the solutions to Equation I and all the points representing the solutions to Equation II .

ג. **(1)** Find how many vertices the polygon has.

(2) Are all the polygon's vertices located on one circle whose center is at the origin? Explain your answer.

We know that the area of the polygon is 62 .

ד. Find the value of r .

/Continued on page 4/

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the function $f(x) = e^x - \frac{1}{1-e^x}$. Function $f(x)$ is defined for all $x \neq 0$.

א. **(1)** Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

(2) Does the graph of function $f(x)$ have intersection points with the axes? Explain your answer.

(3) Find the coordinates of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.

ב. Sketch a graph of function $f(x)$.

Consider the function $g(x) = -x + \ell n(e^x - 1)$.

ג. **(1)** Find the domain [תחום ההגדרה] of function $g(x)$.

(2) Show that $g'(x) = f(x) - e^x$ for all x in the domain of function $g(x)$.

A perpendicular is drawn from the extremum of function $f(x)$ to the x-axis.

ד. Find the area bounded by the graph of function $f(x)$, the x-axis, the perpendicular, and the line $x = \ell n(b)$. $b > 2$ is a parameter. Express your answer in terms of b .

We know that the area you found in Item ד equals $b - 2 + \ell n(1.5)$.

ה. Find the value of b .

/Continued on page 5/

5. Consider the function $f(x)=\frac{a+\ell nx}{x}$, defined on the domain $x>0$. a is a parameter.

א. In Sub-items (1)–(3), express your answers in terms of a , if necessary.

(1) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

(2) Find the coordinates of the intersection point of the graph of function $f(x)$ with the x-axis.

(3) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.

ב. Sketch a graph of function $f(x)$.

Consider the function $g(x)=\frac{1}{x}$, defined on the domain $x\neq 0$.

We know that the graph of function $f(x)$ and the graph of function $g(x)$ intersect at a point where $x=e^2$.

ג. Find the value of a .

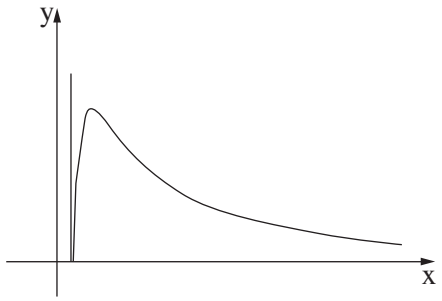
Substitute the value of a that you found, and answer Items ד–ה.

Consider the function $h(x)$, defined on the domain $x>0$. The graph of function $h(x)$ passes through the point $(e^5, 2.5)$.

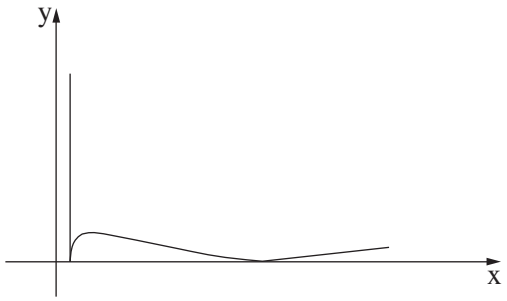
The derivative of function $h(x)$ satisfies $h'(x)=f(x)-g(x)$.

ד. Find function $h(x)$.

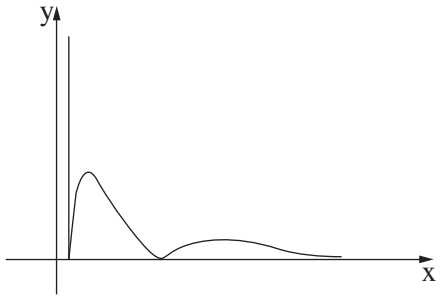
ה. Determine which of the following graphs I–IV represents the function $|h(x)|$. Explain your answer.



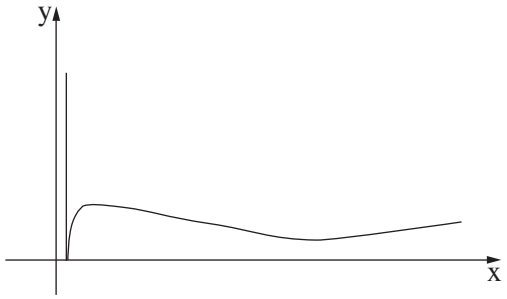
II



I



IV



III

Good Luck!

בהצלחה!