

Note: This exam has special instructions.
Answer the questions according to these instructions.

שים לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions

- א. Duration of the exam: Two and a quarter hours.
- ב. Exam structure and breakdown of points:
This exam has two parts with a total of five questions.
- Part One – Sequences, Spatial Trigonometry
- Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions
- Answer three questions with at least one question from each part — $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.

If you use any draft paper outside the answer booklet your exam may be disqualified.

הוראות

- א. משך הבחינה: שתיים ורבע.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון – סדרות, טריגונומטריה במרחב
פרק שני – גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה
- יש לענות על שלוש שאלות, לפחות על שאלה אחת מכל פרק — $3 \times 33\frac{1}{3} = 100$ נקודות.

- ג. חומר עזר מותר בשימוש:
- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
 - (2) שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
 - (3) דפי נוסחאות (מצורפים).
 - (3) מילון עברי–לועזי / לועזי–עברי.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון גם כאשר החישובים מתבצעים בעזרת מחשבון.
- יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
- חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש

טיוטה.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5 with at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Sequences, Spatial Trigonometry

Sequences

1. Consider a geometric sequence A of 10 terms, whose common ratio is q .
The sixth term in the sequence is 81 times the second term in the sequence.
א. Find the two values of q .

We know that the sum of the two middle terms in Sequence A is 1,944, and that all the sequence's terms are positive.

- ב. Find the first term in the sequence.

Consider the arithmetic sequence B . The sum of Sequence A is 11 times greater than the sum of Sequence B .

- ג. Find the sum of Sequence B .

Sequence B has 33 terms.

We know that the second term in Sequence B is 46 times greater than the sequence's common difference.

- ד. Find the common difference of Sequence B .

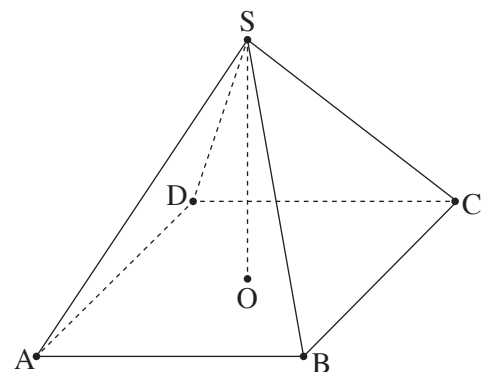
Spatial Trigonometry

2. The figure on the right describes a right pyramid $SABCD$ whose base, $ABCD$, is a rectangle.
We know that the length of the pyramid's lateral edge is 1.25 times greater than the pyramid's height, SO .

- א. Calculate the size of the angle between a lateral edge of the pyramid and the pyramid's base.

We know that the area of Triangle SOC is 54, and that the length of Edge AB is equal to the length of the pyramid's height.

- ב. (1) Find the length of SO .
(2) Find the volume of Pyramid $SABCD$.



Point P is located on the pyramid's height such that Segment CP bisects the angle OCS .

- ג. Calculate the length of Segment PO .

Segments were drawn from Point P to Vertices A , B , and D to form a triangular right pyramid $PABD$.

- ד. How many times greater is the volume of Pyramid $SABCD$ than the volume of

Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

3. Consider the function $f(x) = 3 \cos(2x) - a$, a is a parameter.

Function $f(x)$ is defined on the domain $-\pi \leq x \leq \pi$.

- א. Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are. Express in terms of a , if necessary.

We know that all the minima of function $f(x)$ are located on the line $y = a - 11$.

- ב. Find the value of a .

Substitute $a = 4$ in function $f(x)$, and answer Items ג–ד.

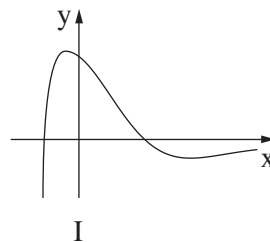
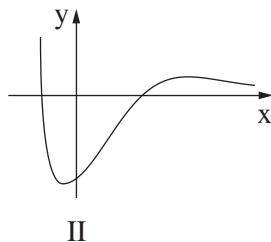
- ג. Sketch a graph of function $f(x)$.

- ד. Calculate the area of the region bounded by the graph of function $f(x)$ and by the line $y = a - 11$ on the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

4. The function $f(x)$ and its derivative function $f'(x)$ are defined for all x .

Function $f(x)$ has only one maximum; the x -coordinate of this maximum is positive.

One of the two graphs below, I and II, describes the derivative function $f'(x)$.



- א. Determine which of the graphs describes the derivative function $f'(x)$, and explain your answer.

We know that $f(x) = (x^2 - 2) \cdot e^{(-2x+1)}$.

- ב. (1) Find the coordinates of the intersection points of the graph of function $f(x)$ with the x -axis.
(2) Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
ג. Sketch a graph of function $f(x)$.

Consider the function $g(x) = 7 \cdot e^{(-2x+1)}$, defined for all x .

- ד. (1) Explain why function $g(x)$ is positive for all x .
(2) Find the coordinates of the intersection points of the graph of function $f(x)$ with the graph of function $g(x)$.

Through each point that you found in Item ד, we draw a perpendicular to the x -axis.

- ה. Calculate the area of the region bounded by the graph of function $g(x)$, the x -axis, and the perpendiculars.

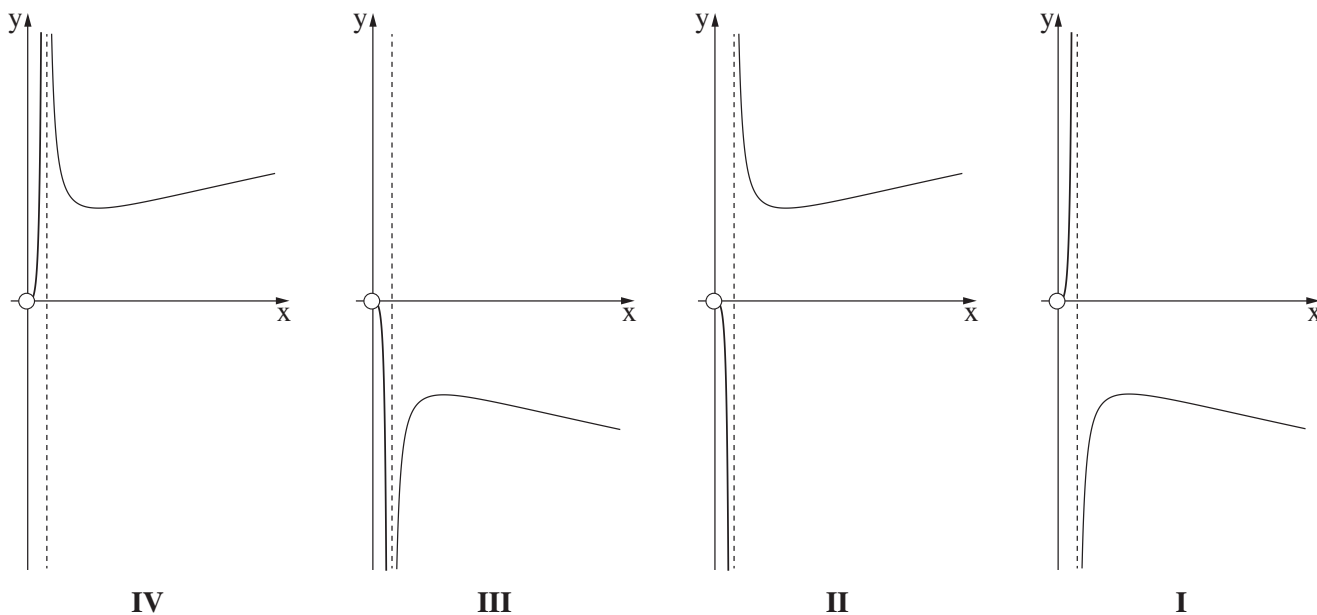
5. Consider function $f(x) = \frac{12x}{(\ln x)^2}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x-axis.
- (3) Does the graph of function $f(x)$ have intersection points with the axes? Explain your answer.
- ב. (1) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.
- (2) Find the intervals on which function $f(x)$ is increasing.
- ג. Determine which of the graphs I–IV at the end of this question describes function $f(x)$.

$g(x)$ is a function whose domain is identical to the domain of function $f(x)$.

The derivative of function $g(x)$ satisfies $g'(x) = f(x) - c$, c is a parameter.

- ד. (1) For $c = 4$, determine how many extrema function $g(x)$ has, and what type of extrema they are (if there are any).
Explain your answer.
- (2) Give a value of c for which function $g(x)$ has three extrema. Explain your answer.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך