The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2025
Exam Number: 35481
Appendix: Formula sheets for 4 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תשפ"ה, 2025
מספר השאלון: 35481
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

Note: This exam has special instructions. Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד – שאלון ראשון

Instructions

א. Duration of the exam: Four hours.

ב. Exam structure and breakdown of points:
This exam consists of three parts with a total of eight questions
Part One – Algebra, Analytical Geometry, and Probability
Part Two – Planar Geometry and Planar Trigonometry
Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions
Answer **five** questions with at least one question from each part — 5×20 = 100 points.

ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

הוראות

א. משך הבחינה: ארבע שעות.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון — אלגברה, גאומטרייה אנליטית והסתברות
פרק שני — גאומטרייה וטריגונומטרייה במישור
פרק שלישי — חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש
יש לענות על **חמש** שאלות, לפחות על שאלה אחת מכל פרק — 5×20 = 100 נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי / לועזי–עברי.

ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל פעולותיכם, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer five of the questions 1–8 with at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Algebra, Analytical Geometry, and Probability

1. The distance between Town A and Town B is 28 km.
On Sunday, Noa cycled from Town A to Town B at a constant speed,
and Yoram walked from Town B to Town A at a constant speed.
They both set off at the same time and met an hour and 10 minutes later.
That day, Yoram reached the midpoint of the route 4 hours after Noa reached the midpoint of the route.

א. Find Noa's cycling speed and Yoram's walking speed.

On Monday, Noa cycled from Town A towards Town B at the same speed as on Sunday, and Yoram walked from Town B towards Town A at the same speed as on Sunday.
That day Noa set off on her bike at 8:00 and Yoram set off on foot 16 minutes later.

ב. At what time did Yoram and Noa meet on Monday?

2. Triangle ABE is inscribed in a circle whose center is M.
AB is a diameter of the circle (see figure).
We know that $M(1,1)$, $B(1,-4)$.

א. Find the coordinates of Vertex A.

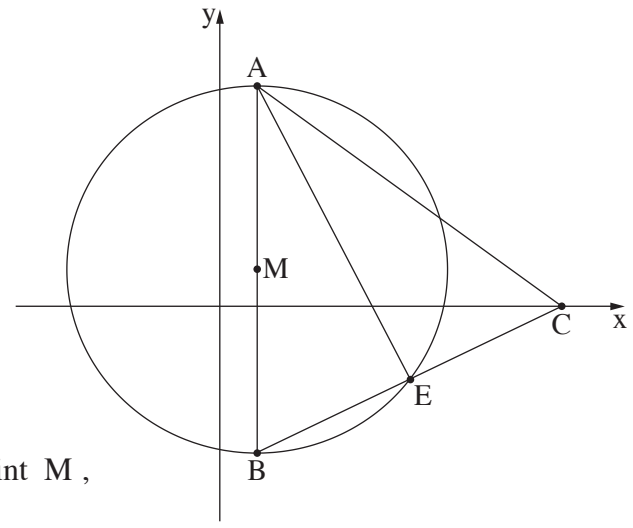
The extension of Side BE intersects the positive part of the x-axis at Point C.
We know that $AB = AC$.

ב. Find the coordinates of Point C.

ג. Find the equation of Line AE.

A line parallel to the x-axis was drawn through Point M, intersecting Side AE at Point D.

ד. Calculate the area of Quadrilateral MDEB.



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3. The 11th grade of a certain school consists of both boys and girls. Some have a driver's license and the rest do not.

25 % of the 11th grade students are boys who do not have a driver's license.

$\frac{1}{3}$ of the 11th grade boys have a driver's license.

א. What percentage of the 11th grade students are boys?

The probability of randomly selecting from the 11th grade a boy with a driver's license is equal to the probability of randomly selecting from the 11th grade a girl with a driver's license.

ב. We know that a student (boy or girl) without a driver's license was randomly selected from the 11th grade. What is the probability that the selected student was a boy?

There are 69 boys in the 11th grade.

ג. Find the total number of students in this grade.

26 new students (boys and girls) joined the 11th grade.

Two students were randomly selected from the 11th grade one after another (sampling without replacement [הוצאה ללא החזרה]).

The probability that both selected students are boys is $\frac{10}{77}$.

ד. Find how many boys joined the 11th grade.

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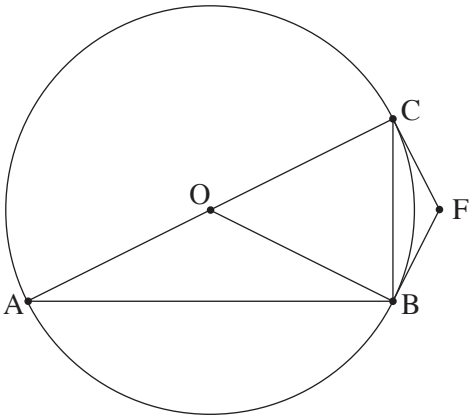
Part Two – Planar Geometry and Planar Trigonometry

4. Consider a circle whose center is O . Triangle ABC is inscribed in the circle such that AC is a diameter of the circle.

Tangents to the circle were drawn through Points B and C , meeting at Point F (see figure).



א. Prove that Quadrilateral OCFB is a kite.

ב. Prove that $\sphericalangle CAB = \sphericalangle COF$.

ג. Prove that $\Delta AOB \sim \Delta CFB$.

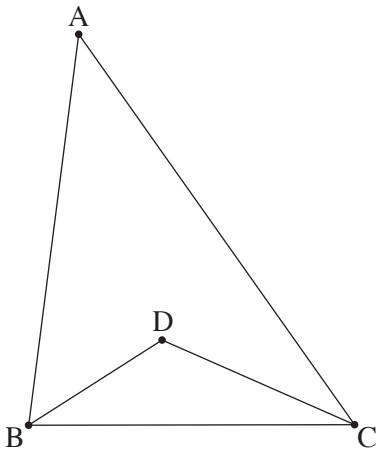
We know that the area of Triangle AOB is 4 times greater than the area of Triangle CFB .

Point E is the intersection point of the diagonals of Kite OCFB .

ד. Prove that $CB = OE$.

5. Consider the acute triangle ABC . Point D is located inside Triangle ABC such that $\sphericalangle BDC = 120^o$ (see figure).

We know that $DB = 6$, $DC = 8$.



א. Find the length of Side BC .

We know that the radius of the circle that circumscribes Triangle ABC is 1.3 times greater than the radius of the circle that circumscribes Triangle DBC .

ב. Find the size of Angle BAC .

We know that $AB = 15$.

ג. Find the length of Side AC .

ד. Calculate the area of Quadrilateral BACD .

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Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x)=\frac{x^2+x+a}{x}$. $a\neq 0$ is a parameter.

א. **(1)** Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes (if there are any).

We know that the graph of function $f(x)$ passes through the point $(2, 7.5)$.

ב. Find the value of a .

Substitute $a=9$ and answer items ג–ה.

ג. Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

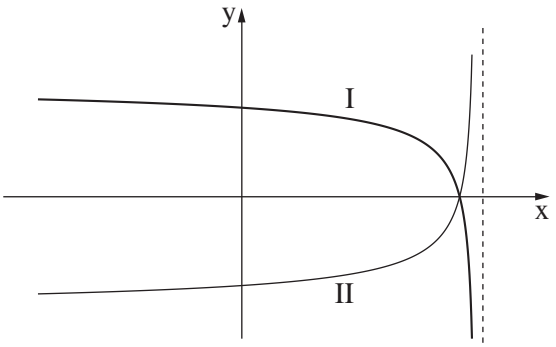
ד. Sketch a graph of function $f(x)$.

$g(x)$ is a function whose domain is identical to the domain of function $f(x)$.
The derivative of function $g(x)$ satisfies $g'(x)=f(x)-11$.

ה. **(1)** Sketch a graph of the derivative function $g'(x)$.

(2) Find the x-coordinates of the extrema of function $g(x)$, and determine what type of extrema they are.

7. Function $f(x)$ is defined on the domain $x\leq 2$, and its derivative function $f'(x)$ is defined on the domain $x<2$.
Function $f(x)$ has only one local extremum [נקודת קיצון פנימית], which is a maximum.
The figure on the right shows two graphs, I and II, one of which describes the derivative function $f'(x)$.



א. Determine which of the graphs describes the derivative function $f'(x)$, and explain your answer.

We know that: $f(x)=4x+2\sqrt{8-4x}$.

ב. Find the coordinates of <u>all</u> the extrema of function $f(x)$, and determine what type of extrema they are.

The graph of function $f(x)$ intersects the x-axis at only one point, on the negative part of the axis.

ג. Find the coordinates of the intersection point of the graph of function $f(x)$ with the x-axis.

ד. Sketch a graph of function $f(x)$.

Consider the function $g(x)=-3\cdot f'(x)$.

ה. Find the area bounded by the graph of function $g(x)$, the x-axis, and the y-axis.

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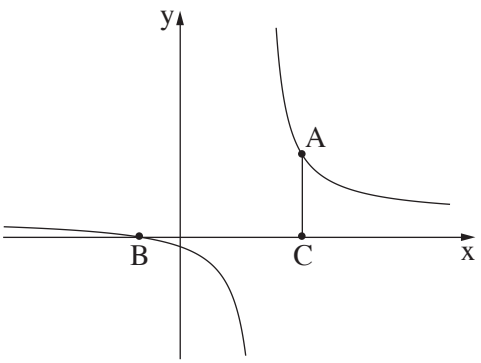
8. The figure on the right describes the graph of function $f(x)=\frac{x+2}{x-4}$.

Point A is located on the graph of function f(x) in the first quadrant.

Point C is located on the x-axis such that Segment AC is parallel to the y-axis.

Point B is the intersection point of the graph of function f(x) with the x-axis.



א. **(1)** Find the domain [תחום ההגדרה] of function f(x) .

(2) Find the equations of the asymptotes of function f(x) that are perpendicular to the axes.

ב. Find the coordinates of Point B .

ג. **(1)** Find the coordinates of Point A for which the area of Triangle ABC is minimum.

(2) Find the minimum area of Triangle ABC .

Good Luck!

בהצלחה!