

Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד - שאלון ראשון

Instructions

הוראות

א. Duration of the exam: Four hours.

א. משך הבחינה: ארבע שעות.

ב. Exam structure and breakdown of points:

This exam consists of three parts, with a total of eight questions.

Part One – Statistics and Probability

Part Two – Geometry

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer **five** questions, at least **one** question from each part – $5 \times 20 = 100$ points.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

פרק ראשון – סטטיסטיקה והסתברות

פרק שני – גאומטריה

פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

יש לענות על **חמש** שאלות, לפחות על שאלה אחת מכל פרק

– $5 \times 20 = 100$ נקודות.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי / לועזי–עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

חסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

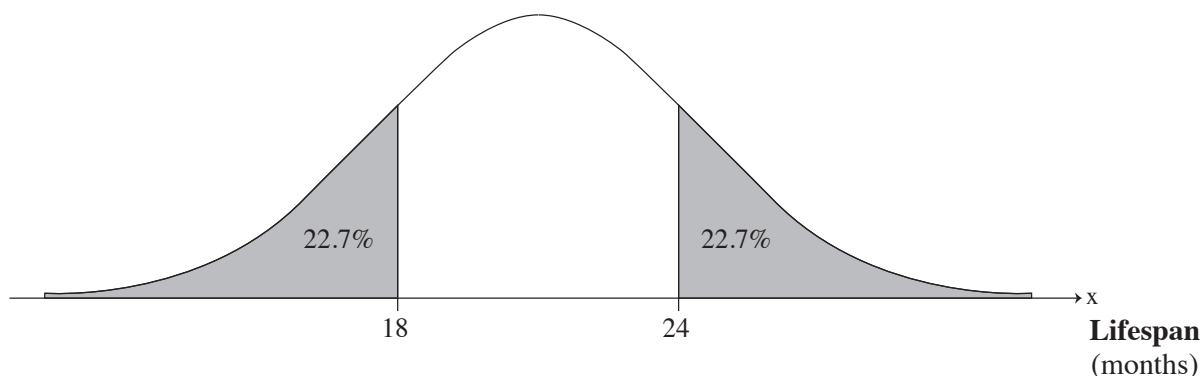
Answer five of the questions 1–8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Statistics and Probability

1. The lifespan of phones produced by a certain factory has a normal distribution.

Below is the normal distribution graph for the lifespan of the phones and some of the lifespan data (in months).



- א. Find the average lifespan of the phones and the standard deviation.

3.5 % of the phones – those with the shortest lifespan – are considered defective.

- ב. What is the longest lifespan of a phone that is considered defective?

One month the factory produced 2,000 phones.

- ג. According to the normal distribution graph, how many phones produced that month had a lifespan of more than 26 months?

Engineers managed to increase the lifespan of all the phones produced in the factory by a factor of 1.5 [1.5 פי].

- ד. (1) What is the phones' new average lifespan and what is the new standard deviation, after the increase?
(2) Find the percentage of phones that have a lifespan of less than 18 months, after the increase.

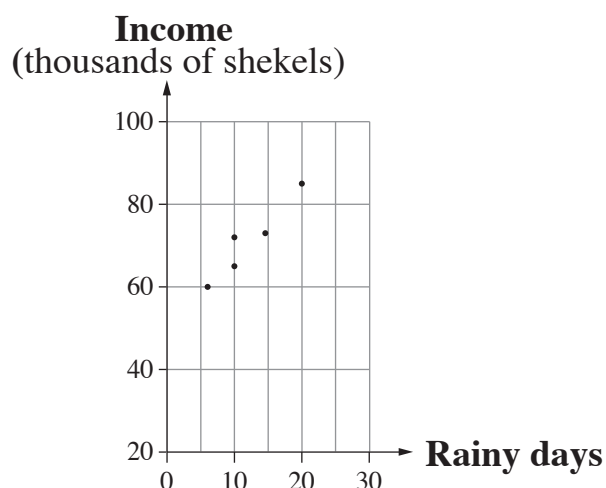
2. In a certain town, over the course of 5 months, the manager of Pharmacy A monitored the relationship between the number of rainy days in a month (Variable x) and the pharmacy's monthly income for that same month (Variable y).

The table below presents the data that the manager collected.

Month	Rainy days (Variable x)	Income (thousands of shekels) (Variable y)
November	6	60
December	10	72
January	20	85
February	14	73
March	10	65

- א. Find the average number of rainy days per month and the standard deviation.

Below is a scatter diagram that describes y as a function of x .



Pharmacy A's average monthly income (in thousands of shekels) is 71, and the standard deviation is 8.46.

One of the numbers 0.959, 1, -0.959 is the correlation coefficient r [מקדם המתאם].

- א. Determine which of them is the correlation coefficient r , and explain your answer.
- א. (1) Find the slope of the regression line for predicting y according to x .
- (2) Find the equation of the regression line for predicting y according to x .
- ב. Based on the regression line, what is the predicted income for a month in which the number of rainy days is 15?

Over the course of the same 5 months, the manager of Pharmacy B in the same town also monitored the relationship between the number of rainy days in a month and the pharmacy's monthly income.

The manager found that every month Pharmacy B's income was 20 thousand shekels less than Pharmacy A's income.

- ב. Write the regression line equation for predicting Pharmacy B's monthly income (in thousands of shekels) according to the number of rainy days in a month.

3. The 11th grade of a certain school consists of both boys and girls. Some have a driver's license and the rest do not.

25 % of the 11th grade students are boys who do not have a driver's license.

$\frac{1}{3}$ of the 11th grade boys have a driver's license.

א. What percentage of the 11th grade students are boys?

The probability of randomly selecting from the 11th grade a boy with a driver's license is equal to the probability of randomly selecting from the 11th grade a girl with a driver's license.

ב. We know that a student (boy or girl) without a driver's license was randomly selected from the 11th grade. What is the probability that the selected student was a boy?

There are 69 boys in the 11th grade of this school.

ג. Find the total number of students in this grade.

26 new students (boys and girls) joined the 11th grade of this school.

Two students were randomly selected from the 11th grade one after another (sampling without replacement [הוצאה ללא החזרה]).

The probability that both selected students are boys is $\frac{10}{77}$.

ד. Find how many boys joined the 11th grade.

Part Two – Geometry

4. ABC is a right triangle ($\angle ABC = 90^\circ$).

Side AB intersects the y -axis at Point P .

We know that Line CP is parallel to the x -axis.

Point E is located on Line CP such that AE is parallel to the y -axis (see figure).

א. Prove that $\triangle AEP \sim \triangle CBP$.

We know that the similarity ratio between

Triangle AEP and Triangle CBP is $\frac{3}{5}$ and that $CP = 12.5$.

ב. Find the length of Segment AP .

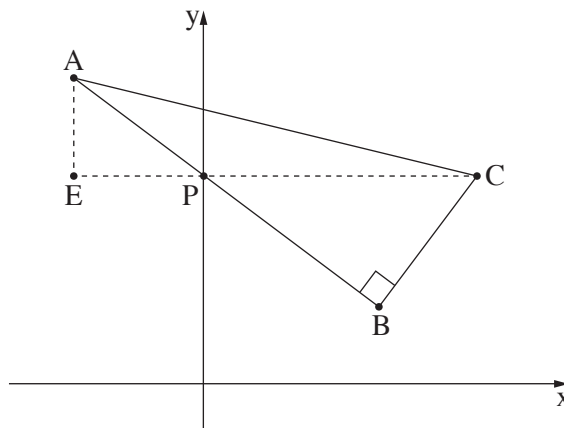
We know that $A(-6, 14)$ and that the y -coordinate of Point P is smaller than 14.

ג. Find the coordinates of Points P and C .

ד. (1) Find the equation of Side AB .

(2) Find the coordinates of Vertex B .

(3) Find the area of Quadrilateral $AEBC$.



5. Triangle ABC is inscribed in a circle whose center is M , as shown in the figure.

We know that $M(-5, 0)$, $C(-15, 0)$.

א. Find the equation of the circle.

We know that the equation of Side AB is $y = 2x$.

ב. Find the coordinates of Vertex A and Vertex B .

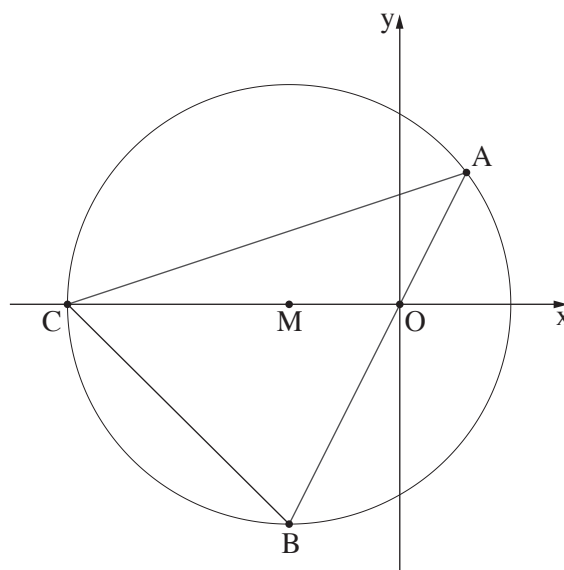
Point O is the origin.

ג. Find the size of Angle ACO and the size of Angle AOC .

ד. Find the area of Triangle ABC .

Point E is located on Side AC such that the area of Triangle EAB is 64.

ה. Find the length of Segment AE .



Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = \frac{x^2 + x + a}{x}$. $a \neq 0$ is a parameter.

א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes (if there are any).

We know that the graph of function $f(x)$ passes through the point $(2, 7.5)$.

ב. Find the value of a .

Substitute $a = 9$ and answer items ג-ה.

ג. Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

ד. Sketch a graph of function $f(x)$.

$g(x)$ is a function whose domain is identical to the domain of function $f(x)$.

The derivative of function $g(x)$ satisfies $g'(x) = f(x) - 11$.

ה. (1) Sketch a graph of the derivative function $g'(x)$.

(2) Find the x -coordinates of the extrema of function $g(x)$, and determine what type of extrema they are.

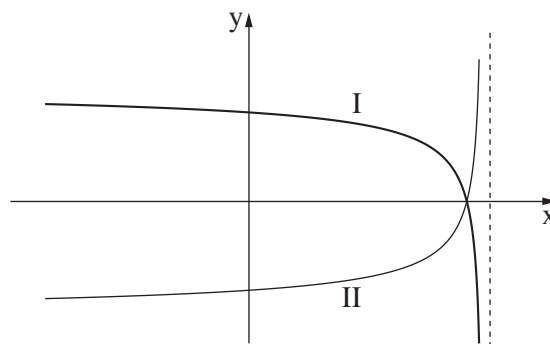
7. Function $f(x)$ is defined on the domain $x \leq 2$, and its derivative function $f'(x)$ is defined on the domain $x < 2$.

Function $f(x)$ has only one local extremum

[נקודת קיצון פנימית], which is a maximum.

The figure on the right shows two graphs, I and II, one of which describes the derivative function $f'(x)$.

א. Determine which of the graphs describes the derivative function $f'(x)$, and explain your answer.



We know that: $f(x) = 4x + 2\sqrt{8 - 4x}$.

ב. Find the coordinates of all the extrema of function $f(x)$, and determine what type of extrema they are.

The graph of function $f(x)$ intersects the x -axis at only one point, on the negative part of the axis.

ג. Find the coordinates of the intersection point of the graph of function $f(x)$ with the x -axis.

ד. Sketch a graph of function $f(x)$.

Consider the function $g(x) = -3 \cdot f'(x)$.

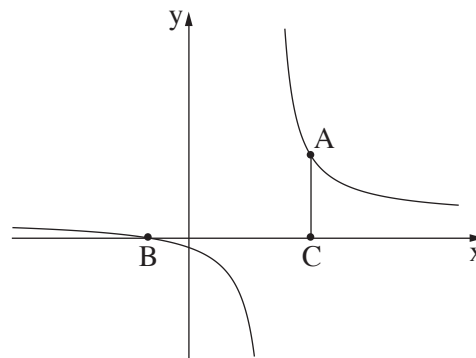
ה. Find the area bounded by the graph of function $g(x)$, the x -axis, and the y -axis.

8. The figure on the right describes the graph of function $f(x) = \frac{x+2}{x-4}$.

Point A is located on the graph of function $f(x)$ in the first quadrant.

Point C is located on the x-axis such that Segment AC is parallel to the y-axis.

Point B is the intersection point of the graph of function $f(x)$ with the x-axis.



- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
- ב. Find the coordinates of Point B.
- ג. (1) Find the coordinates of Point A for which the area of Triangle ABC is minimum.
- (2) Find the minimum area of Triangle ABC.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך