The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2024, ***Moed Bet***
Exam Number: 35582
Appendix: Formula sheets for 5 points
English Translation (3)

Note: This exam has special instructions. Answer the questions according to these instructions.

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Three hours.

ב. Exam structure and breakdown of points:

There are two parts in this exam, with a total of five questions.

Part One: Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Power Functions, Exponential and Logarithmic Functions

Answer three questions of your choice — $3\times33\frac{1}{3}=100$ points.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"ד, 2024, **מועד ב**
מספר השאלון: 35582
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

שימו לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות

א. משך הבחינה: שלוש שעות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטרייה אנליטית, וקטורים, טריגונומטרייה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות חזקה, פונקציות מעריכיות ולוגריתמיות

יש לענות על שלוש שאלות לבחירתכם — $3\times33\frac{1}{3}=100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

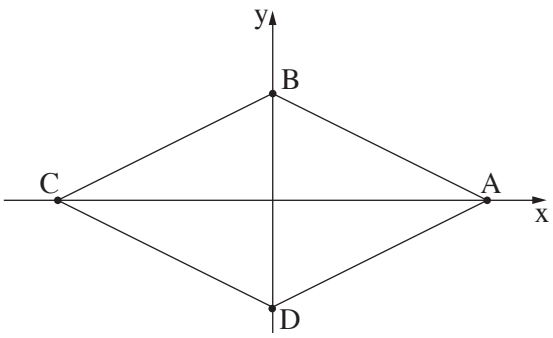
Questions

Answer three of the questions 1–5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider Rhombus ABCD . The diagonals of the rhombus are located on the axes, as shown in the figure.



We know that: the length of Diagonal AC is 10 ;
the distance of each of the sides of the rhombus from the origin is $\sqrt{5}$.

א. Find the equation of Side AB .

A circle is inscribed within the rhombus.

ב. Find the equation of the circle.

Point M is the tangent point of the circle and the rhombus in the first quadrant.

ג. Find the coordinates of Point M .

From Point M a perpendicular is dropped to the x-axis, intersecting it at Point $K(a, 0)$.

On the line $x = -a$ we mark Point E and draw a line through it parallel to the x-axis.

The parallel line intersects the perpendicular bisector of Segment EK at Point G .

ד. Show that the locus [המקום הגאומטרי] of all Points G obtained in this way is located on a parabola, and find the equation of this parabola.

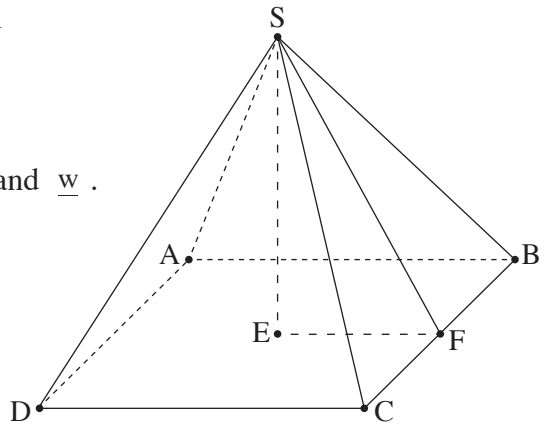
Point N is located in the first quadrant, on the parabola whose equation you found.

The x-coordinate of Point N is 5 .

ה. Find the equations of the two circles whose centers are at Point N and that are tangent to the circle inscribed in the rhombus.

/Continued on page 3/

2. Consider a pyramid SABCD with a square base ABCD.
Point E is the intersection point of the base diagonals, and
Point F is the midpoint of Edge BC.
We denote: $\overrightarrow{SB} = \underline{w}$, $\overrightarrow{SF} = \underline{v}$, $\overrightarrow{SE} = \underline{u}$.



א. Express the vectors $\overrightarrow{BC}$ and $\overrightarrow{DC}$ in terms of $\underline{u}$, $\underline{v}$, and $\underline{w}$.

Segment SE is the altitude [גובה] of the pyramid.
We know that $|\underline{u}| = 7$.

ב. Find the value of $\underline{u} \cdot \underline{w}$.

We know that $\overrightarrow{BA} = (-3, 4, 5)$.

ג. Find the size of the angle between SB and the pyramid base.

We know that: $E(0, 4, 5)$; the plane of Base ABCD is parallel to the z-axis.

ד. Find the equation of the plane on which the pyramid base rests.

We know that the x-coordinate of Vertex B is 3.

ה. Find the coordinates of Vertex B.

3. **א.** **(1)** Find the solutions to the equation: $z^6 + 64i = 0$ (z is a complex number).

(2) Find the four solutions to the equation: $\frac{z^6 + 64i}{z^2 - 4i} = 0$ (z is a complex number).

The solutions that you found in Sub-Item א(2) represent vertices of a quadrilateral on the Gauss plane.

ב. Find the area of the quadrilateral.

We rotate the quadrilateral about the origin at an angle α in a counter-clockwise direction, $0^o < \alpha < 90^o$.

ג. What is the value of the product of all the numbers that represent the vertices of the quadrilateral for $\alpha = 45^o$? Explain your answer.

ד. **(1)** Find the two values of α for which the product of all the numbers that represent the vertices of the quadrilateral after the rotation will be a pure imaginary number.

(2) For each of the values of α that you found, what is the value of that product?

/Continued on page 4/

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the functions $k(x) = xe^x$ and $m(x) = 2e^x - 1$, defined for all x.

א. **(1)** Find the equation of the asymptote that is perpendicular to the y-axis for each of the functions $k(x)$ and $m(x)$.

(2) Find the increasing and decreasing intervals of each of the functions $k(x)$ and $m(x)$ (if there are any).

(3) Find the coordinates of the intersection point of each of the graphs of the functions $k(x)$ and $m(x)$ with the y-axis.

The graphs of the functions $k(x)$ and $m(x)$ intersect at exactly two points, one point where $x = a$, and another point where $x = b$, $b > a$.

ב. Sketch a graph of function $k(x)$ and a graph of function $m(x)$ on the same coordinate system.

Consider the function $f(x) = \frac{e^x - 1}{x - 1}$, defined for all $x \neq 1$.

ג. **(1)** Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

(2) Find the coordinates of the intersection point of the graph of function $f(x)$ with the x-axis.

ד. Explain why the equation $f'(x) = 0$ has exactly two solutions.

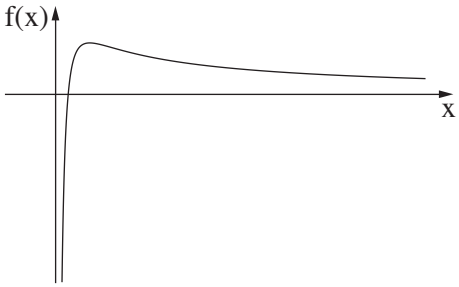
ה. **(1)** Explain why $1 < b$.

(2) Express the increasing and decreasing intervals of function $f(x)$ in terms of a and b.

ו. Sketch a graph of function $f(x)$.

/Continued on page 5/

5. The graph on the right describes the function $f(x)=\frac{2\ell n(x)-1}{x}$, defined on the domain $x>0$.



א. Find the coordinates of the intersection point of the graph of function $f(x)$ with the x-axis.

Consider the function $g(x)$, defined on the domain $x>0$, which satisfies: $g'(x)=f(x)$.
The y-coordinate of the extremum [נקודת הקיצון] of function $g(x)$ is: $-\frac{1}{4}$.

ב. **(1)** Find function $g(x)$.

(2) Find the coordinates of the intersection points of the graph of function $g(x)$ with the x-axis.

(3) Sketch a graph of function $g(x)$.

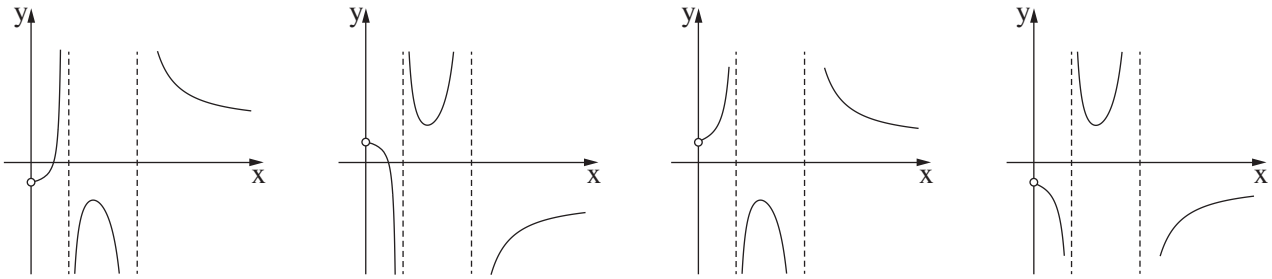
Consider function $h(x)$ defined such that: $h(x)=1+\frac{b}{g(x)}$ where b is a parameter greater than $\frac{1}{4}$.

ג. **(1)** Find the domain [תחום ההגדרה] of function $h(x)$.

(2) Determine whether the graph of function $h(x)$ intersects the x-axis. Explain your answer.

ד. Find the coordinates of the extremum of function $h(x)$, and determine its type. Express your answer in terms of b , if necessary.

ה. Determine which of the graphs I-IV below describes function $h(x)$.



IV **III** **II** **I**

Good Luck!

בהצלחה!