

Note: This exam has special instructions. Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

Mathematics

4 Points – 2nd Exam

מתמטיקה

4 יחידות לימוד – שאלון שני

Instructions

הוראות

- א. Duration of the exam: Two and a half hours
- ב. Exam structure and breakdown of points:
This exam has two parts, with a total of five questions.
Part One – Sequences, Spatial Trigonometry
Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions
Answer any three questions you choose
 $- 3 \times 33\frac{1}{3} = 100$ points.

- א. משך הבחינה: שתיים וחצי שעות.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון – סדרות, טריגונומטריה במרחב
פרק שני – גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה
- יש לענות על שלוש שאלות לבחירתכם – $3 \times 33\frac{1}{3} = 100$ נקודות.

- ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.
- ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

- ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי / לועזי–עברי.
- ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.
If you use any draft paper outside the answer booklet, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your booklet will be graded.

Part One – Sequences, Spatial Trigonometry

Sequences

1. Sequence A is an arithmetic sequence whose terms are: $a_1, a_2, a_3 \dots$, and which contains 23 terms.

We know that $a_{12} = 7$, the common difference of the sequence is 5.

א. Find a_1 .

ב. Find the sum of the terms in the odd positions in the sequence.

Sequence B is an arithmetic sequence whose terms are: $b_1, b_2, b_3 \dots$, and which also contains 23 terms. The first term in the sequence is 3.

Let d denote the common difference of Sequence B.

Using all the terms in Sequences A and B we construct a new arithmetic sequence whose terms are: $a_1 + b_1, a_2 + b_2, a_3 + b_3, \dots$

- ג. (1) Find the first term of the new sequence.
(2) Express the common difference of the new sequence in terms of d .
(3) We know that the sum of all the terms in the new sequence is 3,013.
Find d .

Spatial Trigonometry

2. The figure to the right shows a rectangular cuboid ABCDA'B'C'D' whose base ABCD is a square.

The diagonals of Base ABCD intersect at Point O.

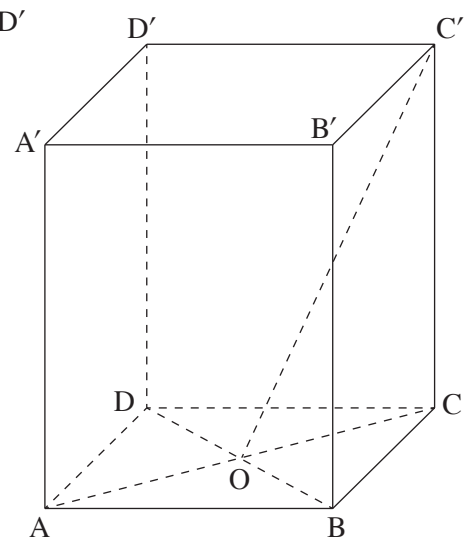
We know that the height of the cuboid is equal to the diagonal of the base.

- א. Find the size of the angle between Segment OC' and Base ABCD.

We know that the length of Segment OC' is $\sqrt{80}$.

- ב. (1) Calculate the area of Triangle BC'D.
(2) Calculate the lateral surface area [שטח המעטפת] of the cuboid.

- ג. Point M is the midpoint of Segment A'D', and Point P is the midpoint of Segment AD.
(1) Calculate the length of Segment BP.
(2) Find the size of the angle between Segment BM and Base ABCD.



Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

3. Consider the function $f(x) = (\cos x)^2 - 2 \sin x - 2$, defined on the domain $0 \leq x \leq 2\pi$.
- א. Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
 - ב. Sketch a graph of function $f(x)$.
 - ג. Find the positive interval [תחום החיוביות] of the derivative function $f'(x)$.
 - ד. Sketch a graph of function $-f(x)$.

Consider the function $g(x) = -f(x) - 1$.

- ה. Determine how many intersection points the graph of function $g(x)$ has with the x-axis. Explain your answer.

4. Consider the function $f(x) = \frac{e^{(ax-1)}}{x^2}$ where a is a parameter.
- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
 - (2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x-axis.
 - (3) Explain why function $f(x)$ is positive for all x in its domain.

We know that the point $\left(-1, \frac{1}{e^3}\right)$ is located on the graph of function $f(x)$.

- ב. Find the value of a .

Substitute $a = 2$ in function $f(x)$, and answer Items ג–ד below.

- ג. (1) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.
- (2) Sketch a graph of function $f(x)$.

Consider the function $g(x) = f(x) + k$ where k is a parameter.

We know that the line $y = -2e$ and the graph of function $g(x)$ share exactly two common points.

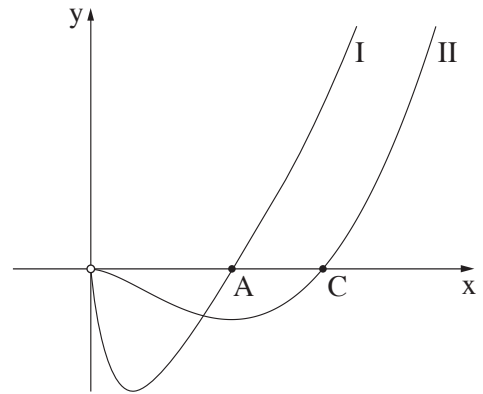
- ד. Find the value of k .

5. Two graphs are described in the figure to the right, I and II .

One of the graphs describes the function $f(x)$, and the other describes the derivative function $f'(x)$.

א. Determine which of the graphs I , II describes the derivative function $f'(x)$.

Explain your answer.



Given: $f(x) = x^2 \cdot (2\ln(x) - 1)$.

ב. (1) Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and use the graph to determine what type of extremum it is.

Point A is the point of intersection of Graph I with the x-axis, and

Point C is the point of intersection of Graph II with the x-axis.

ג. Find the length of Segment AC .

A line parallel to the y-axis is drawn through Point C .

ד. Find the area bounded by Graph I , the line parallel to the y-axis, and the x-axis (the area in the first quadrant).

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך