

Note: This exam has special instructions.
Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics

3 Points – 3rd Exam

Instructions

- א. Duration of the exam: Two and a half hours
- ב. Exam structure and breakdown of points:
This exam has six questions on the following topics:
Algebra, Differential and Integral Calculus.
Answer four questions – each question is worth 28 points.
The maximum number of points you can earn is 100.
- ג. Material that may be used during the exam:
- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
 - (2) Formula sheets (attached).
 - (3) A Hebrew–foreign language/foreign language–Hebrew dictionary.
- ד. Special instructions:
- (1) Do not copy the question, but do indicate the number of the question you are answering.
 - (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to disqualification of your exam.

Write in the answer booklet only. Write "טייטה" at the top of every page you use for rough work.
If you use pages outside the answer booklet for rough work, your exam may be disqualified.

מתמטיקה

3 יחידות לימוד – שאלון שלישי

הוראות

- א. משך הבחינה: שעתיים וחצי.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שש שאלות בנושאים –
אלגברה, חשבון דיפרנציאלי ואינטגרלי.
יש לענות על ארבע שאלות – לכל שאלה 28 נקודות.
סך הנקודות לא יעלה על 100.
- ג. חומר עזר מותר בשימוש:
- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
 - (2) דפי נוסחאות (מצורפים).
 - (3) מילון עברי–לועזי/לועזי–עברי.
- ד. הוראות מיוחדות:
- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
 - (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
 - יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
 - חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer four of the questions 1–6 (each question – 28 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. The price of an adult entrance ticket to the swimming pool is 12 shekels higher than the price of a child entrance ticket.

Raheli bought 5 adult entrance tickets and 19 child entrance tickets.

Let x be the price of a child entrance ticket.

- א. Express in terms of x the total amount that Raheli paid for the pool entrance tickets.

Ofir also bought entrance tickets to the swimming pool.

He bought 7 adult entrance tickets and 30 child entrance tickets.

Ofir was given a 15% discount on the price of the child entrance tickets (the price of an adult entrance ticket remained the same).

The total amount that Raheli and Ofir paid together for all the tickets was 2,460.5 shekels.

- ב. Find x .
- ג. By what percentage was the amount that Ofir paid for all the tickets he bought greater than the amount that Raheli paid for all the tickets she bought? (Your answer should be correct to two decimal places.)

2. Vertex A in Quadrilateral OABC is located on the y-axis,
and Vertex C is located on the x-axis.

Vertex O is the origin (see figure).

We know that Diagonal AC is perpendicular to
Diagonal OB.

The equation of Diagonal AC is $y = -\frac{1}{2}x + 15$.

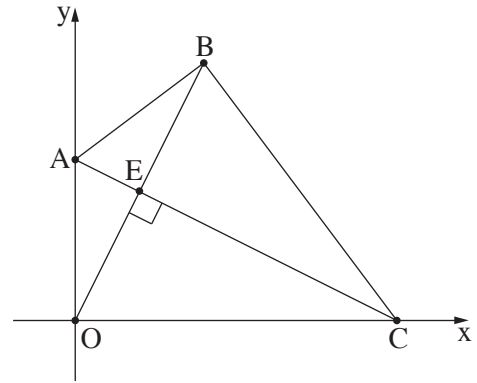
- א. Find the coordinates of Vertex A and Vertex C.
- ב. Find the equation of Diagonal OB.

Point E is the intersection point of the two diagonals.

- ג. Find the coordinates of Point E.

Point E is the midpoint of Diagonal OB.

- ד. Find the coordinates of Vertex B.
- ה. (1) Prove that Triangle OAB is an isosceles triangle.
(2) Find the perimeter of Quadrilateral OABC.



3. Consider a circle whose center is M and equation is $(x + 6)^2 + (y - 8)^2 = 208$.

The circle intersects the x-axis at Points A and B, as shown in the figure.

- א. Find the coordinates of the center of the circle, M.
- ב. Find the coordinates of Points A and B.

Point C is located on the circle such that AC is a diameter of the circle.

- ג. (1) Find the coordinates of Point C.
(2) Find the equation of Line AC.

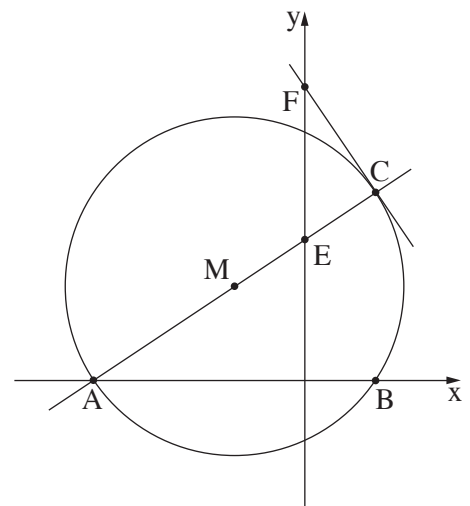
A tangent to the circle was drawn through Point C.

The tangent intersects the y-axis at Point F.

- ד. (1) Find the equation of the tangent.
(2) Find the coordinates of Point F.

Line AC intersects the y-axis at Point E.

- ה. Find the area of Triangle FCE.



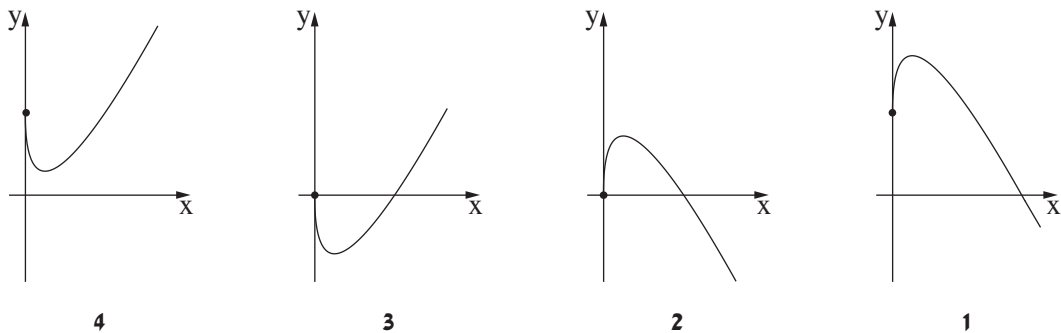
Differential and Integral Calculus

4. Consider function $f(x) = 4x - 12\sqrt{x} + 13$.

- א. Find the domain [תחום ההגדרה] of function $f(x)$.
- ב. Find the coordinates of the intersection point of the graph of function $f(x)$ with the y-axis.
- ג. (1) Find the coordinates of the local extremum [נקודת הקיצון הפנימית] of function $f(x)$, and determine what type of extremum it is.
(2) Find the interval on which function $f(x)$ is increasing.
- ד. Determine which of the graphs 1–4 at the end of the question describes function $f(x)$.

A tangent of slope 2 was drawn to the graph of function $f(x)$.

- ה. Find the coordinates of the tangent point.



5. Consider the function $f(x) = -x^2 + 10x - 8$ and the line $y = -x + 10$.

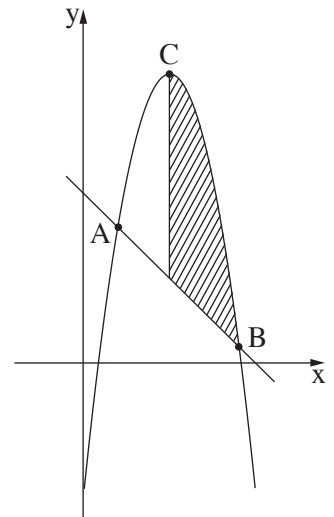
Points A and B are the intersection points of the graph of function $f(x)$ and the line, as shown in the figure.

Point C is the maximum of function $f(x)$.

- א. Find the coordinates of Points A and B.
- ב. Find the coordinates of Point C.

A line was drawn through Point C, parallel to the y-axis.

- ג. Calculate the area of the shaded region in the figure:
the region bounded by the graph of function $f(x)$, the line given at the beginning of this question, and the line parallel to the y-axis.



6. In Triangle ABC, Segment BE is the height to Side AC (see figure).

We know that the length of Segment EC is 3 times greater than the length of Segment AE, and that $AC + BE = 20$.

Let $AE = x$.

- א. (1) Express the length of Side AC in terms of x .
- (2) Express the length of the height BE in terms of x .
- ב. Find the value of x for which the area of Triangle ABC is maximum (greatest).

