The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2024
Exam Number: 35582
Appendix: Formula sheets for 5 points
English Translation (3)

Note: This exam has special instructions. Answer the questions according to these instructions.

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Three hours.

ב. Exam structure and breakdown of points:

There are two parts in this exam, with a total of five questions.

Part One: Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Power Functions, Exponential and Logarithmic Functions

Answer three questions of your choice — $3\times33\frac{1}{3}=100$ points.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"ד, 2024
מספר השאלון: 35582
נספח: דפי נוסחאות ל־5 יחידות לימוד
תרגום לאנגלית (3)

שימו לב: בבחינה זו יש הנחיות מיוחדות. יש לענות על השאלות על פי הנחיות אלה.

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות

א. משך הבחינה: שלוש שעות.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטרייה אנליטית, וקטורים, טריגונומטרייה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות חזקה, פונקציות מעריכיות ולוגריתמיות

יש לענות על שלוש שאלות לבחירתכם — $3\times33\frac{1}{3}=100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer three of the questions 1–5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider Circle I of center K, whose equation is $(x-3)^2+(y-4)^2=9$.
Consider Circle II of center L, whose equation is $(x+5)^2+(y-2)^2=1$.
Point A is located on Circle I and Point B is located on Circle II.
A tangent to Circle I is drawn through Point A, and a tangent to Circle II is drawn through Point B. The two tangents intersect at Point M.

א. Show that the locus [המקום הגאומטרי] of all the points M that satisfy $MA = MB$ is a line; find the equation of this line.

ב. **(1)** Show that the line whose equation you found in Item א and Line KL are perpendicular to each other.

(2) Is there a point M for which $ML = MK$? Explain your answer.

We know that for one point M, located above Line KL, the area of Triangle KLM is 9.

ג. Find the coordinates of Point M.

Point M that you found in Item ג is located on the parabola $y^2 = 2px$ where p is a parameter.

ד. Find the equation of the tangent to the parabola at Point M.

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2. Consider two lines, ℓ_1 and ℓ_2. The parametric form [הצגה פרמטרית] of each of the lines is shown below.

$\ell_1 : \underline{x} = t(-1, 3, 0)$

$\ell_2 : \underline{x} = (1, -3, 0) + m(0, k, 1)$

where k is a parameter.

Point A is the intersection point of the two lines, ℓ_1 and ℓ_2.

א. Find the coordinates of Point A.

Let α be the angle between the lines ℓ_1 and ℓ_2.

ב. Find the value of k for which $\cos\alpha = \frac{3\sqrt{2}}{5}$ is satisfied.

Substitute $k = 2$, and answer Items ג–ה.

The plane π contains the lines ℓ_1 and ℓ_2.

ג. Find the equation of the plane π.

Point B is located on the line ℓ_2, and Point O is the origin.

Triangle AOB is an isosceles triangle, $AO = AB$.

ד. Explain why Triangle AOB is on the plane π.

A perpendicular to the plane π is drawn up from Point A, and Point S is marked on the perpendicular.

We know that the volume of Pyramid $SAOB$ is $\frac{14\sqrt{2}}{3}$.

ה. Find the coordinates of Point S (both possibilities).

3. In Rhomboid $ABCD$, Vertex A is in the first quadrant and Vertex B is in the second quadrant on the Gauss plane.

The diagonals of the rhomboid intersect at the origin.

The complex number z represents Vertex A.

We know that $BD = 2AC$.

א. Express the numbers that represent Vertices B, C, and D in terms of z.

Let $z = r(\cos\theta + i\sin\theta)$.

ב. Answer Sub-Items (1) and (2). Express your answers in terms of r and θ, if necessary.

(1) Give a polar representation of the four reciprocals $(\frac{1}{z}, ...)$ of the numbers that represent the vertices of the rhomboid.

(2) The four numbers you found in Sub-Item ב(1) represent the vertices of a quadrilateral. Find the area of the quadrilateral formed by these vertices.

ג. Consider the equation $w^{11} = \overline{w}$; w is a non-zero complex number.
Find the sum of the equation's 12 solutions.

The solutions to the equation in Item ג represent the vertices of a polygon whose area is the same as the area of the quadrilateral you found in Sub-Item ב(2).

ד. Find the value of r.

/Continued on page 4/

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the function $f(x)=\frac{e^x-b}{(e^x-4)^2}$ where b is a positive parameter, $b\neq 4$.

א. Answer Sub-Items (1)–(3). If necessary, express your answers in terms of b.

(1) Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

(3) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.

We know that function $f(x)$ has an extremum at the point where $x=\ell n(12)$.

ב. Find the value of b, and the type of extremum.

Substitute the value of b that you found in function $f(x)$, and answer Items ג–ו.

Consider the function $g(x)=\frac{1}{f(x)}$.

ג. **(1)** Find the domain of function $g(x)$.

(2) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.

ד. Sketch a graph of function $f(x)$ and a graph of function $g(x)$ on the same coordinate system.

ה. What is the y-coordinate of the intersection points of the graph of function $f(x)$ with the graph of function $g(x)$? Explain your answer.

ו. Determine whether the value of $\int_{-2}^{-1}(f(x)-g(x))dx$ is smaller or greater than $1\frac{1}{2}$. Explain your answer.

5. Consider the function $f(x)=x\left((\ell n(x))^2-2\ell n(x)+2\right)$, defined on the domain $x>0$.

א. Find the intervals on which function $f(x)$ increases and decreases (if there are any).

ב. Find the coordinates of the inflection point [נקודת פיתול] of function $f(x)$.

ג. Find the positive and negative intervals of function $f(x)$ (if there are any).

ד. **(1)** Sketch a graph of function $f(x)$.

(2) Sketch a graph of the derivative function $f'(x)$.

Consider the functions $g(x)=(\ell n(x))^2-4$, $h(x)=\frac{f(x)}{x^2}$, defined on the domain $x>0$.

ה. Find the area bounded by the graph of function $g(x)$ and the x-axis.

Perpendiculars to the x-axis were drawn through the intersection points of the graph of function $g(x)$ with the x-axis.

ו. Find the area bounded by the graph of function $g(x)$, the graph of function $h(x)$, and the perpendiculars.

Good Luck!

בהצלחה!

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זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך