The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2024
Exam Number: 35482
Appendix: Formula sheets for 4 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"ד, 2024
מספר השאלון: 35482
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

Note: This exam has special instructions. Answer the questions according to these instructions.

שים לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה.

Mathematics
4 Points – 2nd Exam

מתמטיקה
4 יחידות לימוד – שאלון שני

Instructions

א. Duration of the exam: Two and a half hours

ב. Exam structure and breakdown of points:
This exam has two parts, with a total of five questions.

Part One – Sequences, Spatial Trigonometry
Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

Answer any three questions you choose – $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.
If you use any draft paper outside the answer booklet, your exam may be disqualified.

Good Luck!

הוראות

א. משך הבחינה: שעתיים וחצי.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון – סדרות, טריגונומטרייה במרחב
פרק שני – גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה

יש לענות על שלוש שאלות, לבחירתכם – $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי / לועזי-עברי.

ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer three of the questions 1–5 (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your booklet will be graded.

Part One – Sequences, Spatial Trigonometry

Sequences

1. A certain train has 11 cars. On Sunday, the number of passengers in each car was greater by a fixed number than the number of passengers in the car in front of it.

The number of passengers in the last car was 3 times greater than the number of passengers in the first car.

That day, the total number of passengers on the train was 220 .

א. Find how many passengers were in the first car.

ב. Find how many passengers were in the middle car.

ג. Find the total number of passengers in the odd-numbered cars (the first, third, fifth car, and so on).

On Monday, the number of passengers on the train was 2 times greater than the number of passengers on Sunday, so extra cars were added to the train.

On Monday, the number of passengers in each car was greater by 3 than the number of passengers in the car in front of it.

That day, the first car contained 5 passengers.

ד. Find the number of extra cars added to the train on Monday.

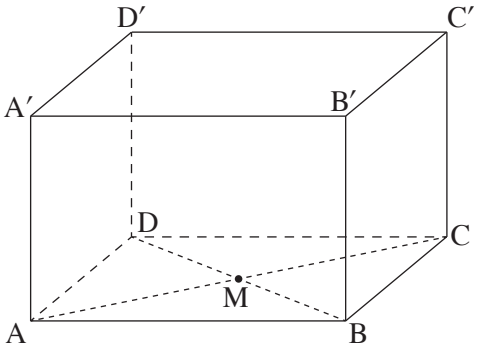
Spatial Trigonometry

2. The figure to the right shows the rectangular cuboid $ABCDA'B'C'D'$.

The diagonals of the base $ABCD$ intersect at Point M .

We know that $AB = 12$, $BC = 5$.

We also know that the size of the angle between the segment MC' and the base $ABCD$ is 50^o .



א. Find the length of the cuboid's height.

ב. Find the surface area of the cuboid.

The diagonals of the base $A'B'C'D'$ intersect at Point M' .

Point P is located on the segment MM' , such that $AP = 1.5MP$.

Segments were drawn between Point P and the vertices of the base $ABCD$, such that a right pyramid $PABCD$ was formed.

ג. Find the size of the angle between a lateral edge of the pyramid and the base $ABCD$.

ד. Find the volume of the pyramid .

/Continued on page 3/

Part Two – Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

3. Consider the function $f(x) = a - \frac{1}{2}(\sin x)^2$, a is a parameter.

Function $f(x)$ is defined on the domain $0 \le x \le 2\pi$.

א. Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are. Express in terms of a , if necessary.

The line $y = 2$ is tangent to the graph of function $f(x)$.

ב. Find the two possible values of a .

Take the smaller value of a that you found, substitute it in function $f(x)$, and answer Items ג–ד.

ג. Sketch a graph of function $f(x)$.

Consider function $g(x)$, which satisfies $g'(x) = f(x)$.

Functions $f(x)$ and $g(x)$ are defined on the same domain.

ד. Consider the two statements I and II below. Determine whether each statement is true or false. Explain your answers.

I. Function $g(x)$ has 3 local extrema [נקודות קיצון פנימיות].

II. Function $g(x)$ is increasing on the interval $0 < x < 2\pi$.

4. Consider function $f(x) = \frac{e^{2x}}{e^x - 3}$.

א. **(1)** Find the domain [תחום ההגדרה] of function $f(x)$.

(2) Find the equation of the asymptote of function $f(x)$ that is perpendicular to the x-axis.

ב. Find the coordinates of the points of intersection of the graph of function $f(x)$ with the axes (if there are any).

ג. Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.

ד. Find the intervals on which function $f(x)$ is increasing or decreasing.

ה. Sketch a graph of function $f(x)$.

Consider function $g(x)$, which satisfies $g(x) = -f(x) + 15$.

Functions $f(x)$ and $g(x)$ are defined on the same domain.

ו. **(1)** Find the coordinates of the extremum of function $g(x)$, and determine what type of extremum it is.

(2) How many points of intersection does the graph of function $g(x)$ have with the x-axis? Explain your answer.

/Continued on page 4/

5. Consider function $f(x) = ax \cdot \ell n(2x)$, a is a parameter.

א. Find the domain [תחום ההגדרה] of function $f(x)$.

We know that the slope of the tangent to the graph of function $f(x)$ at the point at which $x = \frac{e}{2}$ is 6 .

ב. Find a .

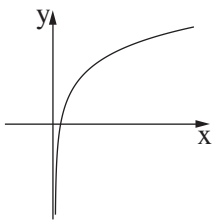
Substitute $a = 3$ in function $f(x)$, and answer Items ג–ו below.

ג. Find the coordinates of the point of intersection of the graph of function $f(x)$ with the x-axis.

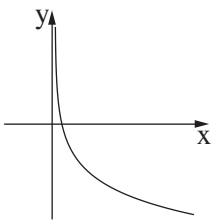
ד. Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.

ה. Determine which of the graphs I–IV at the end of the question describes function $f(x)$ and which graph describes the derivative function $f'(x)$. Explain your answers.

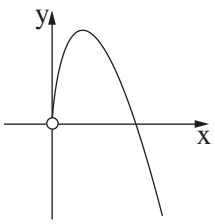
ו. Calculate the area bounded by the graph of the derivative function $f'(x)$, the line $x = e$, and the x-axis.



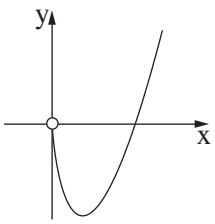
IV



III



II



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Good Luck!

בהצלחה!