The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Winter 2024
Exam Number: 35481
Appendix: Formula sheets for 4 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: חורף תשפ"ד, 2024
מספר השאלון: 35481
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

Note: There are special instructions in this exam. Please answer the exam according to these instructions.

Mathematics

4 Points – 1st Exam

מתמטיקה

4 יחידות לימוד – שאלון ראשון

Instructions

א. Duration of the exam: Four and a quarter hours.

ב. Exam structure and breakdown of points:
This exam consists of three parts, with a total of eight questions.
Part One – Algebra, Analytical Geometry, Probability
Part Two – Planar Geometry and Planar Trigonometry
Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer four questions, at least one question from each part – $4 \times 25 = 100$ points.

ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

הוראות

א. משך הבחינה: ארבע שעות ורבע.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון – אלגברה, גאומטרייה אנליטית, הסתברות
פרק שני – גאומטרייה וטריגונומטרייה במישור
פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

יש לענות על ארבע שאלות, לפחות על שאלה אחת מכל פרק – $4 \times 25 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי / לועזי–עברי.

ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל פעולותיכם, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer four of the questions 1–8, at least one question from each part (each question – 25 points).

Note: If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Part One – Algebra, Analytical Geometry, Probability

1. Town A is located 12 km from Town B .

Town C is located midway between the two (see figure).

Itamar and Nir set off on a walk at 8:00 .

Itamar walked at a constant speed from Town A to Town B .

Nir walked from Town A to Town B , and as soon as he reached Town B walked back to Town C.

Nir's speed as he walked to Town B was greater than Itamar's speed by 2 km/h.

Nir's speed as he walked back from Town B (to Town C) was equal to Itamar's speed.

Itamar reached Town B half an hour before Nir reached Town C .

א. Find Itamar's walking speed, if we know that his walking speed was less than 5 km/h.

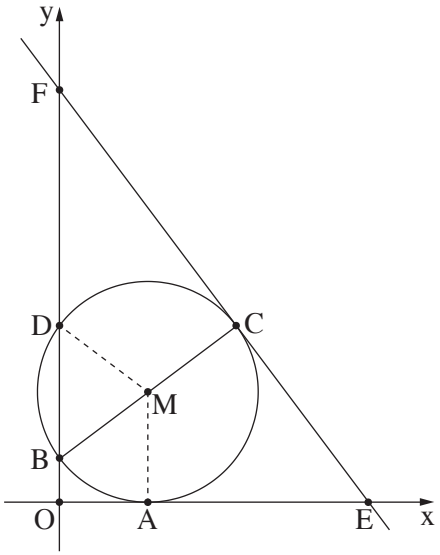
ב. At what time did Nir reach Town B ?

As he was walking back to Town C Nir met Itamar.

ג. At what time did Nir and Itamar meet?

/Continued on page 3/

2. A circle of center M is tangent to the x-axis at Point A . Point O is the origin.
The circle intersects the y-axis at Points B and D , as shown in the figure.



Given: $A(4\,,0)$, $D(0\,,8)$.

א. **(1)** Find the coordinates of Point M .

(2) Find the equation of the circle.

(3) Find the coordinates of Point B .

Point C is located on the circle such that BC is a diameter.
A tangent to the circle is drawn through Point C , intersecting the x-axis at Point E and the y-axis at Point F .

ב. Find the equation of the tangent.

ג. Is Point C the center of the circle circumscribing Triangle EFO ? Explain your answer.

3. Hanan is playing darts. There are only two possible results in this game: hit or miss.
The probability that Hanan will hit the target on his first attempt is $\frac{3}{5}$.
The probability that Hanan will hit the target on his second attempt depends on the result of his first attempt:
If Hanan hits the target on his first attempt, the probability that he will hit it on his second attempt is $\frac{2}{3}$.
If Hanan misses the target on his first attempt, the probability that he will hit it on his second attempt is $\frac{7}{16}$.
Hanan has two tries at hitting the target.

א. What is the probability that Hanan missed the target on his first attempt and hit it on his second attempt?

ב. **(1)** What is the probability that Hanan hit the target at least once?

(2) We know that Hanan hit the target at least once.
What is the probability that he hit the target exactly once?

Danny also has two tries at hitting the target.
The probability that Danny will hit the target on each of his attempts is p .
We know that the probability that Danny will hit the target exactly once is the same as the probability that Hanan will hit the target exactly once.

ג. Find p (both possibilities).

/Continued on page 4/

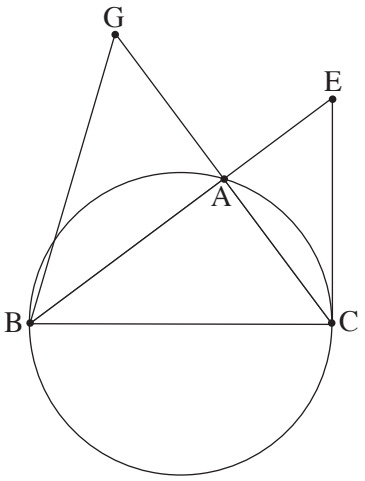
Part Two – Planar Geometry and Planar Trigonometry

4. Triangle ABC is inscribed in a circle. Side BC is a diameter of the circle.

Point G is located on the extension of Side CA, as shown in the figure.

A tangent to the circle is drawn through Point C, intersecting the extension of Side BA at Point E.

We know that $AC = AG$.



א. Prove that $BG = BC$.

ב. Prove that $\sphericalangle ECA = \sphericalangle ABG$.

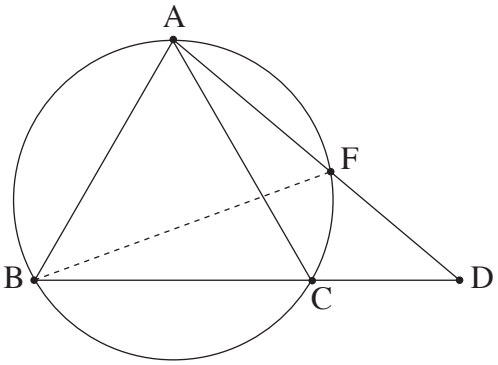
ג. Prove that $\Delta ACE \sim \Delta ABG$.

We know that $AE \cdot AB = 20.25$.

ד. Find the length of Segment AC.

5. Triangle ABC is an <u>equilateral</u> triangle inscribed in a circle of radius 19.

Point D is located on the extension of Side BC, as shown in the figure.



א. Find the length of Side AC.

We know that $CD = 18$.

ב. Find the length of Segment AD.

ג. Find the size of Angle CAD.

Point F is the point at which Segment AD intersects the circle.

ד. Find the length of Chord BF.

ה. **(1)** What is the size of Angle FBC ?

(2) Find the area of Triangle FBD.

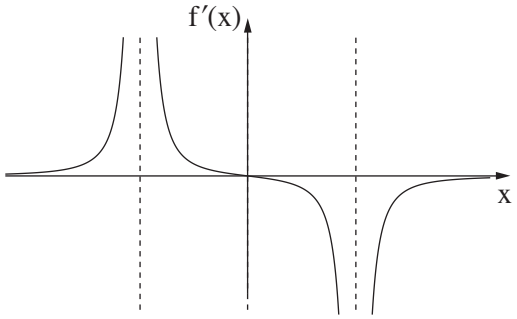
/Continued on page 5/

Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. The function $f(x)$ is defined on the domain [תחום ההגדרה] $x \neq \pm 4$.

The figure below describes the graph of the derivative function $f'(x)$, which is defined on the same domain.

The graph of the derivative function $f'(x)$ intersects the x-axis only at the point $(0\,,0)$.



א. Find the x-coordinate of the extremum [נקודת קיצון] of function $f(x)$, and determine what type of extremum it is.

ב. Find the intervals on which function $f(x)$ is increasing and decreasing.

We know that function $f(x)$ has a horizontal asymptote whose equation is $y = 2$.

One of the expressions I–III below represents function $f(x)$.

I $\dfrac{x^2}{x^2+16}+1$ **II** $\dfrac{x^2}{x^2-16}+2$ **III** $\dfrac{x^2}{x^2-16}+1$

ג. Determine which of the expressions I–III represents function $f(x)$. Explain your answer.

ד. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.

ה. Sketch a graph of function $f(x)$.

ו. Calculate the area bounded by the graph of the derivative function $f'(x)$, the x-axis, and the line $x = 2$.

/Continued on page 6/

7. Consider the function $f(x) = x^2 \cdot \sqrt{-2x+10}$.

א. Find the domain [תחום ההגדרה] of function $f(x)$.

ב. Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.

ג. Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

ד. Sketch a graph of function $f(x)$.

Consider the function $g(x) = f(x) - c$; c is a positive parameter.

The functions $f(x)$ and $g(x)$ are defined on the same domain.

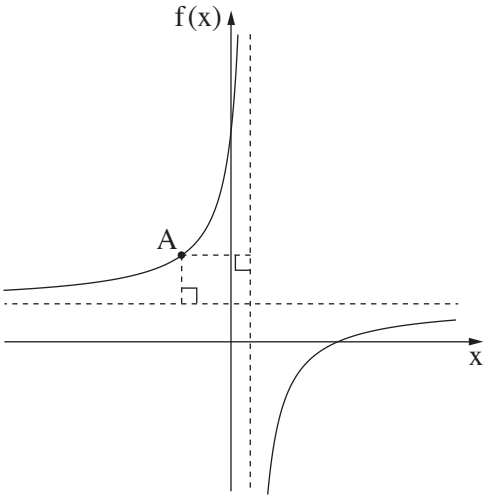
The graph of function $g(x)$ is tangent to the line $y = 20$.

ה. Find the value of c .

8. The figure on the right describes the graph of function $f(x) = \frac{9}{1-x} + 2$, which is defined for all $x \neq 1$.

א. Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.

Perpendiculars to the asytmptotes of function $f(x)$ are dropped from Point A , which is located on the graph of function $f(x)$ in the second quadrant, such that the asymptotes and the perpendiculars form a rectangle.



ב. Find the coordinates of Point A for which the perimeter of the rectangle is at a minimum.

ג. Calculate the <u>area</u> of the rectangle defined by the coordinates of Point A that you found in Item ב.

Good Luck!

בהצלחה!