

Mathematics

5 Points – 2nd Exam

מתמטיקה

5 יחידות לימוד – שאלון שני

Instructions

- א. Duration of the exam: Two hours and fifteen minutes.
- ב. Exam structure and breakdown of points:
There are two parts in this exam, with a total of five questions.
Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers
Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions
Answer three questions, at least one question from each
part – $3 \times 33\frac{1}{3} = 100$ points.
- ג. Material that may be used during the exam:
(1) A non-graphing calculator. You may not use the
programming options of a programmable calculator.
Use of a graphing calculator or programming options
may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language/foreign language–Hebrew
dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of
the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in the answer booklet,
even when the calculations are done with a calculator.
Explain all of your work, including calculations,
in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the
disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft
page. If you use draft paper outside the answer booklet, your exam may be
disqualified.

הוראות

- א. משך הבחינה: שתיים ורבע.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון: גאומטריה אנליטית, וקטורים,
טריגונומטריה במרחב, מספרים מרוכבים
פרק שני: גדילה ודעיכה, פונקציות חזקה,
פונקציות מעריכיות ולוגריתמיות
יש לענות על שלוש שאלות, לפחות על שאלה אחת מכל פרק
– $3 \times 33\frac{1}{3} = 100$ נקודות.
- ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון
שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול
לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש.
יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים
מתבצעים בעזרת מחשבון.
יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה
ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש
טיוטה. כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת
הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers

1. Consider the points $A(0, 24)$ and $B(18, 0)$.

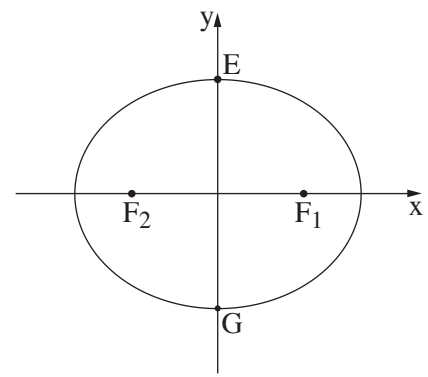
א. Find the equation of the locus [מקום גאומטרי] of the points C that satisfy $AC^2 + BC^2 = 1,250$.

The locus you found in Item א is moved 9 units to the left and 12 units down, so that a new locus is obtained.

The new locus intersects the y -axis at Points E and G (E is above G).

Points F_1 and F_2 are the foci of a canonical ellipse that passes through Points E and G , as shown in the figure.

Given: The distance between the lines EF_1 and GF_2 is 24.



ב. (1) Find the coordinates of Point F_1 .

(2) Find the equation of the ellipse.

Circles are drawn that are tangent to the line EF_1 , the x -axis, and the y -axis.

ג. Find the equations of two such circles, each of which is located in a different quadrant.

2. In Pyramid ABCD, Edge DC is perpendicular to Plane ABC.

Point E is the midpoint of Edge AD.

Point F satisfies $\overrightarrow{DF} = \frac{k}{2} \cdot \overrightarrow{DB} + k \cdot \overrightarrow{DC}$; k is a parameter.

Let: $\overrightarrow{CD} = \underline{w}$, $\overrightarrow{AC} = \underline{v}$, $\overrightarrow{AB} = \underline{u}$.

א. Express $\overrightarrow{EF}$ in terms of $\underline{u}$, $\underline{v}$, $\underline{w}$, and k.

We know that $\overrightarrow{EF}$ is parallel to Plane ABC.

ב. Find the value of k.

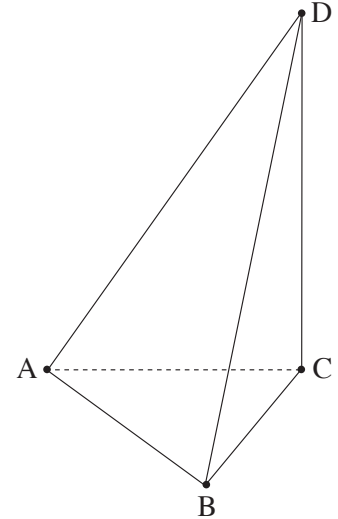
We know that: $A(0, 0, 0)$, $C(0, n, 0)$, $B(p, 3, 0)$; n and p are positive parameters.

$$\underline{u} \cdot \underline{v} = 24, \overrightarrow{BD} = (-4, 5, 12).$$

ג. Find the coordinates of Points C, B, and D.

ד. Find the volume of Pyramid ABCD.

ה. What is the relative position of Line EF and Line AB? Explain your answer.



3. Consider a geometric sequence $z_1, z_2, z_3, \dots$ whose terms are complex numbers and whose common ratio is q.

z_1 is in the first quadrant.

$$\text{Given: } (z_1)^3 = z_3$$

$$-2z_1 = \overline{z_3}$$

א. Prove that $q = -z_1$ or $q = z_1$.

ב. Find z_1 .

Answer Items ג and ד for $q = z_1$.

z_{4n} and z_{4n-2} are two terms in the given geometric sequence (n is a natural number).

ג. Determine for each of the terms whether it is imaginary or real. Explain your answers.

ד. Find the value of the sum $\frac{z_1}{\sqrt{2}} + \frac{z_2}{(\sqrt{2})^2} + \frac{z_3}{(\sqrt{2})^3} + \dots + \frac{z_{64}}{(\sqrt{2})^{64}}$.

Part Two – Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the function $f(x) = \frac{\ln(x) + \ln(a)}{\ln(x) - \ln(a)}$; a is a parameter greater than 1.

In Items א–ג, express your answers in terms of a , if necessary.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
 (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
 (3) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes (if there are any).
 (4) Find the intervals on which function $f(x)$ is decreasing.
 (5) Sketch a graph of function $f(x)$.

Consider the following argument: The equation $f(x) = f'(x)$ has exactly one solution in the interval $x > a$.

- ב. Determine whether this argument is correct or not. Explain your answer.

Consider function $g(x)$, which satisfies $g(x) = \ln(f(x))$.

- ג. (1) Find the domain of function $g(x)$.
 (2) Sketch a graph of function $g(x)$.

Let S be the area bounded by the graph of function $g(x)$, the x -axis, and the lines $x = 2$ and $x = 5$.

We know that $1 < a < 2$.

- ד. Express the value of the integral $\int_2^5 \ln(2 \cdot f(x)) dx$ in terms of S .

5. Consider the function: $f(x) = \frac{e^x}{e^x - 4}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
 (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
 (3) Find the intervals on which function $f(x)$ is increasing and decreasing (if there are any).
 (4) Sketch a graph of function $f(x)$.

- ב. Consider function $g(x)$ which satisfies $g(x) = \frac{1}{f(x)}$. Functions $g(x)$ and $f(x)$ are defined on the same domain.

- (1) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes (if there are any).
 (2) Sketch a graph of function $g(x)$.
 (3) Calculate the area of the region bounded by the graph of function $g(x)$, its horizontal asymptote, and the lines $x = \ln 5$ and $x = \ln 8$.
 ג. Find the coordinates of the intersection point of the graph of function $f(x)$ with the graph of function $g(x)$.

Consider the function $s(x) = \int_x^{\ln 3} (f(t) - g(t)) dt$ that is defined on the domain $x < \ln 3$.

- ד. Find the x -coordinate of the extremum [נקודת קיצון] of function $s(x)$, and determine what type of extremum it is.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
 אין להעתיק או לפרסם אלא ברשות משרד החינוך