The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2023, ***Moed Bet***
Exam Number: 035482
Appendix: Formula sheets for 4 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"ג, 2023, **מועד ב**
מספר השאלון: 035482
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

Mathematics
4 Points – 2nd Exam

מתמטיקה
4 יחידות לימוד – שאלון שני

Instructions

א. Duration of the exam: One hour and 45 minutes

ב. Exam structure and breakdown of points:
This exam has two parts, with a total of five questions.

Part One — Sequences, Spatial Trigonometry
Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

Answer three questions, with at least one question from each part
— $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:
(1) A non-graphing calculator. Do not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
(2) Formula sheets (attached).
(3) A Hebrew–foreign language / foreign language–Hebrew dictionary.

ד. Special instructions:
(1) Do not copy the question, but do indicate the number of the question you are answering.
(2) Start each question on a new page.
Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.
Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page. If you use any draft paper outside the answer booklet your exam may be disqualified.

הוראות

א. משך הבחינה: שעה וארבעים וחמש דקות.

ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שני פרקים, ובהם חמש שאלות.
פרק ראשון — סדרות, טריגונומטרייה במרחב
פרק שני — גדילה ודעיכה, חשבון דיפרנציאלי ואינטגרלי של פונקציות טריגונומטריות, פונקציות מעריכיות ולוגריתמיות ופונקציות חזקה

יש לענות על שלוש שאלות, לפחות על שאלה אחת מכל פרק
— $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי–לועזי / לועזי–עברי.

ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה. תיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5, with at least one question from each part
(each question - $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One — Sequences, Spatial Trigonometry

Sequences

1. Consider the arithmetic sequence a_n of n terms.

We know that $a_1 = -10$; $a_3 = -4$;

the sum of all the terms in the sequence is $10{,}353$.

א. Find n .

Every third term in the sequence a_n was deleted (that is, the terms a_3 , a_6 , ... were deleted).

ב. **(1)** How many terms were deleted from the sequence a_n ?

(2) What is the sum of the terms that were deleted from the sequence a_n ?

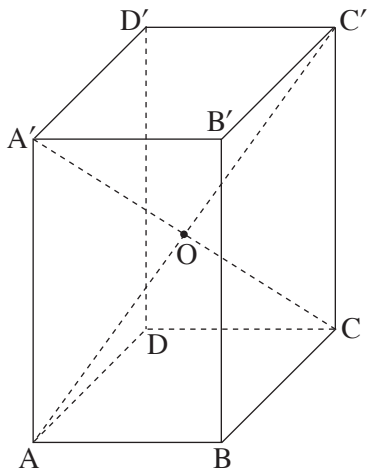
(3) What is the sum of the terms that remained in the sequence a_n , after the others were deleted?

Spatial Trigonometry

2. Consider a cuboid $ABCDA'B'C'D'$ whose base $ABCD$ is a square (see figure).

We know that the diagonal of the cuboid is $\sqrt{3}$ times greater than the diagonal of the base.

א. Find the size of the angle between the diagonal of the cuboid and the base.



We know that the area of the rectangle $ACC'A'$ is $32\sqrt{2}$.

ב. **(1)** Find the side length of the base of the cuboid.

(2) Find the length of the diagonal of the cuboid.

The diagonals of the cuboid, $A'C$ and $C'A$, intersect at Point O .

ג. Find the lateral surface area [שטח המעטפת] of the right pyramid $OABCD$.

/Continued on page 3/

Part Two — Growth and Decay, Differential and Integral Calculus of Trigonometric Functions, Exponential and Logarithmic Functions, and Power Functions

3. Consider the given function $f(x) = 1 + x - \sin(2x)$, which is defined on the domain $0 \le x \le \pi$.

א. Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

ב. Sketch a graph of function $f(x)$.

ג. Find the equation of the tangent to the graph of function $f(x)$ at the point where $x = \frac{\pi}{4}$.

ד. Find the area of the region bounded by the graph of function $f(x)$, the tangent whose equation you found in Item ג, and the y-axis (on the given domain, the tangent meets the graph of the function at the point of tangency only).

4. Consider the given function $f(x) = (2x^2 - 11x + 14) \cdot e^{4-x}$.

א. What is the domain [תחום ההגדרה] of function $f(x)$?

ב. **(1)** Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.

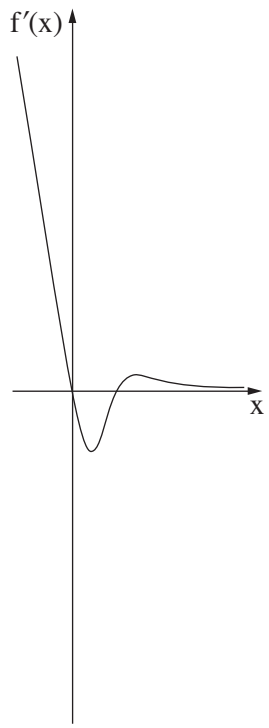
(2) Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.

(3) Sketch a graph of function $f(x)$.

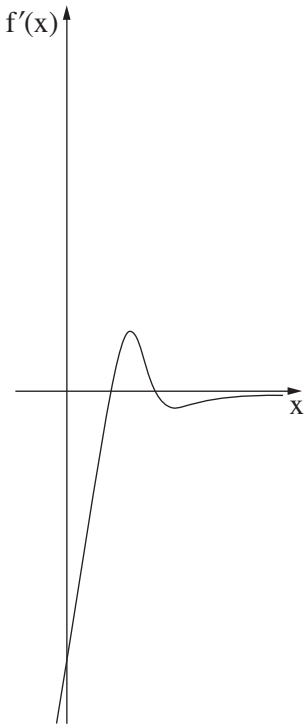
ג. One of the graphs I-III shown at the end of the question describes the derivative function $f'(x)$. Determine which graph it is, and explain your answer.

ד. Determine which values of x satisfy both $f(x) < 0$ and $f'(x) < 0$. Explain your answer.

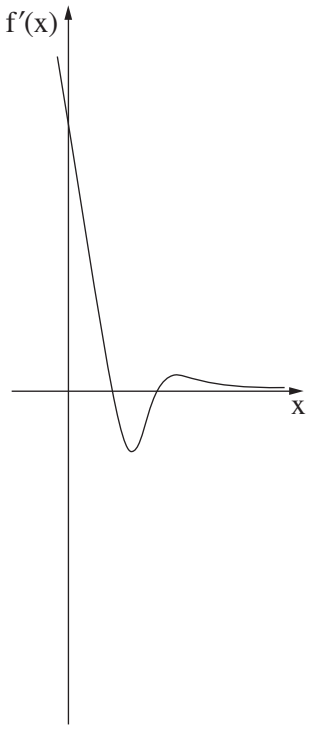
ה. Calculate the area of the region bounded by the graph of the derivative function $f'(x)$ and the x-axis.



III



II



I

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5. Consider the given function $f(x)=\frac{(\ell nx)^2}{4x}$.

א. Find the domain [תחום ההגדרה] of function $f(x)$.

ב. Find the coordinates of the intersection point of the graph of function $f(x)$ with the x-axis.

ג. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.

Consider the given function $g(x)=-f(x)$.

ד. Sketch the graphs of function $f(x)$ and function $g(x)$ on the same coordinate system.

Let S denote the area of the region bounded by the graph of function $f(x)$ and the tangent to the graph of function $f(x)$ at the maximum of the function.

ה. **(1)** Find the equation of the tangent to the graph of function $f(x)$ at the maximum of the function.

(2) Find a value of c for which the area of the region bounded by the graph of function $g(x)$ and the line $y=c$ equals S . Explain your answer.

Good Luck!

בהצלחה!