

Mathematics

4 Points – 1st Exam

Instructions

- א. Duration of the exam: Three and a half hours.
- ב. Exam structure and breakdown of points:
This exam has three parts with a total of eight questions.
Part One: Algebra, Analytical Geometry, Probability
Part Two: Planar Geometry and Planar Trigonometry
Part Three: Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions
Answer **five** questions, at least one question from each part
 $- 5 \times 20 = 100$ points.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all of your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיטה" at the top of each draft page.
If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

4 יחידות לימוד – שאלון ראשון

הוראות

- א. משך הבחינה: שלוש שעות וחצי.
- ב. מבנה השאלון ומפתח ההערכה:
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
פרק ראשון – אלגברה, גאומטריה אנליטית, הסתברות
פרק שני – גאומטריה וטריגונומטריה במישור
פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש
יש לענות על **חמש** שאלות, לפחות על שאלה אחת מכל פרק
 $- 5 \times 20 = 100$ נקודות.
- ג. חומר עזר מותר בשימוש:
(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
(2) דפי נוסחאות (מצורפים).
(3) מילון עברי-לועזי / לועזי-עברי.
- ד. הוראות מיוחדות:
(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
(2) יש להסביר כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל פעולותיכם, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיטה" בראש כל עמוד המשמש טיטה.
כתיבת טיטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer five of the questions 1-8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One — Algebra, Analytical Geometry, Probability

1. Ruti left her house at 7:00 and started walking at a constant speed towards her aunt's house. If Ruti had continued walking at the same speed, she would have reached her aunt's house at exactly 9:00 .

However, a quarter of the way along the route, she stopped to rest for half an hour. She then continued on her way, walking at a constant speed that was greater than her initial speed by 1.2 km/h. Ruti reached her aunt's house at 9:12 .

- א. What was Ruti's walking speed when she left her house?
- ב. What is the distance between Ruti's house and her aunt's house?
- ג. At what time was Ruti halfway along the route?

2. Quadrilateral ABCD in the figure on the right is an isosceles trapezoid, $AD = BC$, $AB \parallel DC$. Point E is the midpoint of Side AB .

We know that: $B(3, 6)$,

the equation of the perpendicular to Side AB that passes through Point E is $y = -2x + 7$.

- א. (1) Find the equation of AB .
- (2) Find the coordinates of Vertex A .

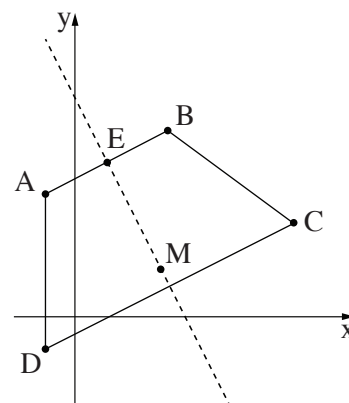
We know that: $C(7, 3)$,

Leg AD is parallel to the y-axis.

- ב. (1) Find the length of Leg BC .
- (2) Find the coordinates of Vertex D .

Point M is located on the given perpendicular and satisfies $AM = DM$.

- ג. Find the coordinates of Point M .
- ד. Find the area of Triangle ADM .



3. In her pencil case, Dana has 25 pencils of the same size but of three different colors:

- 15 blue pencils,
- 4 red pencils,
- 6 yellow pencils.

Dana randomly takes a pencil out of her pencil case. If the pencil is blue or red, she puts it back into the pencil case. If the pencil is yellow, she leaves it out of the pencil case.

Then Dana randomly takes another pencil out of the pencil case.

א. What is the probability that Dana will take out two yellow pencils?

ב. (1) What is the probability that Dana will take out two pencils of **the same color**?

(2) We know that Dana has taken out two pencils of the same color.

What is the probability that she took out two red pencils **or** two yellow pencils?

Dana put all the pencils back into the pencil case. She then took x blue pencils, 3 red pencils, and 2 yellow pencils out of the pencil case and gave them to her brother.

Next, she randomly took two pencils out of her pencil case without replacement.

We know that the probability that Dana took out a yellow pencil followed by a red pencil is $\frac{1}{60}$.

ג. Find x .

Part Two — Planar Geometry and Planar Trigonometry

4. Triangle ABC is inscribed in a circle whose center is O .

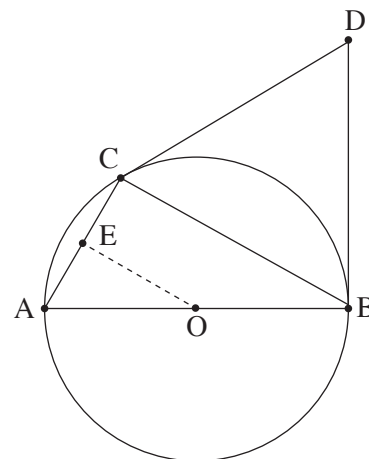
AB is a diameter in the circle.

Point E is the midpoint of Side AC (see figure).

א. (1) Prove that $OE \perp AC$.

(2) Prove that $\triangle ABC \sim \triangle AOE$.

- ב. By what factor [פי כמה] is the area of Quadrilateral $BCEO$ greater than the area of Triangle AOE ? Explain.



We know that AC is equal to the radius of the circle and that the tangents to the circle at Points B and C meet at Point D .

- ג. Prove that Triangle BDC is an equilateral triangle.

We know that the radius of the circle is 4.

- ד. Find the area of Triangle OCD .

5. Triangle ABC is a right triangle, $\angle ACB = 90^\circ$.

Point D is located on the extension of Side AB , as shown in the figure on the right.

We know that $BC = 0.75 \cdot AC$.

- א. Find the size of the angle CBD .

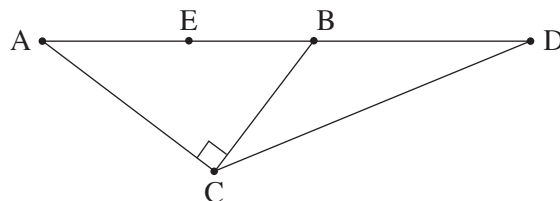
We know that $BD = 8$, $AB = 10$.

- ב. Find the length of DC .

Point E is located on Segment AB and satisfies $DC = DE$.

- ג. Find the area of Triangle EDC .

- ד. Find the radius of the circle that circumscribes Triangle EBC .



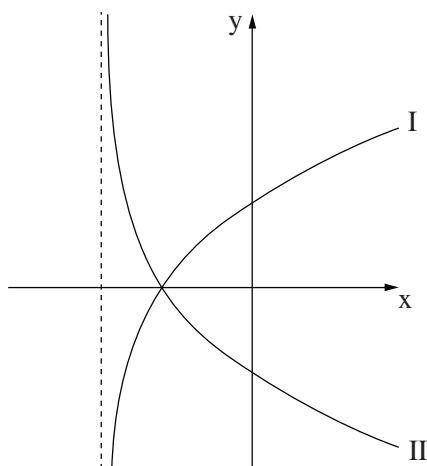
Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = \frac{9 - 4x^2}{1 - x^2}$.
- (1) Find the domain [תחום ההגדרה] of function $f(x)$.
 - (2) Find the asymptotes of function $f(x)$ that are perpendicular to the axes.
 - (3) Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
 - (4) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.
 - (5) Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.
- Sketch a graph of function $f(x)$.
 - Consider function $g(x)$ that satisfies $g'(x) = f(x)$. Functions $f(x)$ and $g(x)$ have the same domain.
Find the x -coordinates of the extrema [נקודות הקיצון] of function $g(x)$, and determine what type of extrema they are. Explain your answer.

7. Consider the function $f(x) = (x - 1) \cdot \sqrt{2x + 10}$.
- Find the domain [תחום ההגדרה] of function $f(x)$.
 - Find the coordinates of the intersection points of the graph of function $f(x)$ with the axes.
 - Find the coordinates of all the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
 - Sketch a graph of function $f(x)$.

Two graphs, I and II, are shown at the end of the question. One of the graphs describes the derivative function $f'(x)$, and the other graph describes the function $g(x) = -f'(x)$.

- Determine which of the graphs I–II describes the derivative function $f'(x)$. Explain your answer.
- Calculate the area of the region bounded by the graph of the derivative function $f'(x)$, the line $x = 3$, and the x -axis.



8. ABC is a right triangle, $\angle ACB = 90^\circ$. The area of Triangle ABC is 72.

M is a point located on Side AC , such that $MC = 2MA$ (see figure).

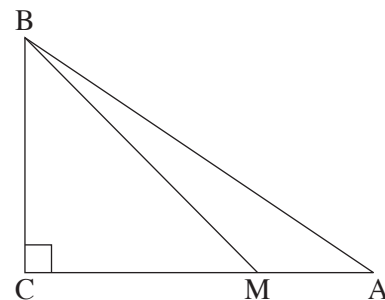
Let x denote the length of Segment MA .

א. Express the length of Side BC in terms of x .

ב. (1) Find the value of x for which the sum of the squares of the distances between Point M and the three vertices of the triangle ($MA^2 + MB^2 + MC^2$) is minimum.

(2) Could the sum $MA^2 + MB^2 + MC^2$ be 300?

Explain your answer.



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך