

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Two hours and fifteen minutes

ב. Exam structure and breakdown of points:

There are two parts in this exam, with a total of five questions.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions

Answer three questions of your choice, at least one from each part
— $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.

Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות

א. משך הבחינה: שתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

יש לענות על שלוש שאלות לבחירתכם,

לפחות שאלה אחת מכל פרק

— $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.

אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש.

יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה מסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או

לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.

כתובת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers

1. Consider an ellipse whose equation is $\frac{x^2}{144} + \frac{y^2}{144 - 4k^2} = 1$; $0 < k < 6$.

Point F_1 is the right focus of the ellipse and Point F_2 is the left focus of the ellipse.

א. Express the coordinates of Points F_1 and F_2 in terms of k .

Point A is located in the first quadrant on a parabola whose equation is canonical and whose focus is at Point F_1 , such that $AF_1 = 10k$.

ב. (1) Express the equation of the directrix of the parabola in terms of k .

(2) Express the coordinates of Point A in terms of k .

AF_1 is a diameter of a circle. The line whose equation is $5x + 12y = 138$ is tangent to this circle.

ג. Find the value of k .

D is a point located on the ellipse.

ד. Determine whether the perimeter of Triangle F_1AF_2 is greater than, smaller than, or equal to the perimeter of Triangle F_1DF_2 . Explain your answer.

2. The figure on the right describes the cube $ABCD A' B' C' D'$.

Let $\overrightarrow{AB} = \underline{u}$, $\overrightarrow{AD} = \underline{v}$, $\overrightarrow{AA'} = \underline{w}$.

א. Prove that the diagonal CA' is perpendicular to the plane $BC'D$.

Point E is the intersection point of the medians (centroid) of Triangle $BC'D$.

א. (1) Express vector $\overrightarrow{CE}$ in terms of $\underline{u}$, $\underline{v}$ and $\underline{w}$.

(2) Prove that Points C , E and A' are located on one line.

Given: $D(0, 0, 0)$, $C(3, 4, 0)$, $A(4, n, p)$; n and p are parameters.

The z -coordinate of Point C' is positive.

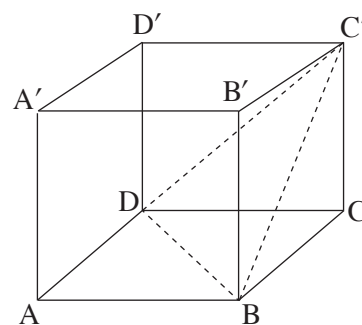
א. (1) Find the coordinates of Point A , and prove that $ABCD$ is located on the plane $z = 0$.

(2) Find the coordinates of Point C' .

ℓ is the line of intersection of the planes $BC'D$ and $BCC'B'$.

א. Find a parametric representation of Line ℓ .

א. Find a parametric representation of the plane that contains Line ℓ and does not intersect the x -axis.



3. Consider the equation $z^3 = \frac{1}{z^3}$; z is a complex number.

We know that the number z_0 is one of the solutions of this equation, and that it is represented by a point located in the fourth quadrant on the Gauss plane.

א. Find the complex number z_0 .

Points A , B and C are represented on the Gauss plane by the complex numbers $d \cdot z_0$, $di \cdot z_0$ and $d \cdot (z_0)^4$ respectively; $d > 0$ is a parameter. -

Given: The area of Triangle ABC is $5d + 6$.

א. Find the value of d .

We define: $w = \left((z_0)^2 - \frac{1}{(z_0)^2} \right) (1 + i)$.

א. Find $|w|$ and the argument (angle) of w .

We know that the number w^n (n is a natural number) is a purely imaginary number, and is located outside the circle that circumscribes Triangle ABC .

א. Find the minimum possible value of n .

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the function $f(x) = (e^x - 1)^n - 4$, defined for any x . n is a natural number greater than or equal to 2.

Answer Item א for even n and for odd n .

- א. (1) Find the equation of the horizontal asymptote of function $f(x)$.
(2) Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are (if there are any).
(3) Sketch a graph of function $f(x)$.

Answer Items ב-ג for $n = 2$.

Consider the function $g(x) = 3e^x - 7$, defined for any x .

- ב. (1) Find the coordinates of the intersection points of the graph of function $f(x)$ with the graph of function $g(x)$.
(2) Calculate the area of the region bounded by the graph of function $f(x)$, and the graph of function $g(x)$.

Consider the function $h(x) = |f(x)|$, defined for any x .

- ג. (1) How many extrema does function $h(x)$ have? Find the coordinates of these extrema, and determine what type of extrema they are.
(2) Find the range of values of k for which the line $y = k$ intersects the graph of function $h(x)$ at 3 points.

5. Consider the function $f(x) = \ln(x) + \frac{1}{x}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
(2) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.
(3) Sketch a graph of function $f(x)$.

Consider the function $g(x) = (x + 1)(1 - \ln(x))$, defined on the same domain as function $f(x)$.

- ב. (1) Find the coordinates of the intersection point of the graph of function $g(x)$ with the x -axis.
(2) Find the intervals on which function $g(x)$ is increasing and the intervals on which it is decreasing (if there are any).
(3) Find the interval of upward concavity $\cup$ and the interval of downward concavity $\cap$ of function $g(x)$.
(4) Sketch a graph of function $g(x)$.

Consider the function $h(x) = \frac{1}{x} \cdot g'(x)$, defined on the same domain as function $g(x)$.

- ג. Calculate the area of the region bounded by the graph of function $h(x)$, the x -axis, and the lines $x = e$ and $x = 1$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך