The State of Israel
Ministry of Education
Exam Type: *Bagrut*
Exam Date: Summer 2023
Exam Number: 035481
Appendix: Formula sheets for 4 points
English Translation (3)

מדינת ישראל
משרד החינוך
סוג הבחינה: בגרות
מועד הבחינה: קיץ תשפ"ג, 2023
מספר השאלון: 035481
נספח: דפי נוסחאות ל־4 יחידות לימוד
תרגום לאנגלית (3)

Mathematics

4 Points – 1st Exam

Instructions

א. Duration of the exam: Three and a half hours

ב. Exam structure and breakdown of points:

This exam has three parts with a total of eight questions.

Part One:	Algebra, Analytical Geometry, Probability
Part Two:	Planar Geometry and Planar Trigonometry
Part Three:	Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

Answer **five** questions,

at least one question from each part — $5\times 20=100$ points.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew–foreign language/ foreign language–Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

(3) Explain all of your work, including calculations, in detail and in a clear and organized fashion.
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טיוטה" at the top of each draft page.
If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

4 יחידות לימוד–שאלון ראשון

הוראות

א. משך הבחינה: שלוש שעות וחצי.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שלושה פרקים, ובהם שמונה שאלות.

פרק ראשון	—	אלגברה, גאומטרייה אנליטית, הסתברות
פרק שני	—	גאומטרייה וטריגונומטרייה במישור
פרק שלישי	—	חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש

יש לענות על **חמש** שאלות,

לפחות שאלה אחת מכל פרק — $5\times 20 = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי / לועזי–עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

(3) יש להסביר את כל פעולותיכם, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.
כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Answer five of the questions 1–8, at least one question from each part (each question – 20 points).

Note: If you answer more than five questions, only the first five answers in your answer booklet will be graded.

Part One – Algebra, Analytical Geometry, Probability

1. On Sunday, Danny and Ariel set out – each from his own house – and started walking towards each other.

Danny walked at a constant speed of 6 km/h, and Ariel walked at a constant speed of 8 km/h.

Danny left his house at 8:00 am, and Ariel left his house at 8:30 am.

They met halfway between their respective houses.

א. Find the distance between Danny's house and Ariel's house.

ב. At what time did Danny and Ariel meet?

On Monday, Danny and Ariel set out – each from his own house – at the same time and started walking towards each other.

Danny walked at a constant speed that was different from his walking speed on Sunday, and Ariel walked at a constant speed that was the same as his walking speed on Sunday.

Danny reached Ariel's house 20 minutes after Ariel reached Danny's house.

ג. What was Danny's walking speed on Monday?

ד. How much time after Danny and Ariel set off on Monday was the distance between them 5 km (before they met)?

/Continued on page 3/

2. The quadrilateral ABCD in the figure on the right is a kite [דלתון].

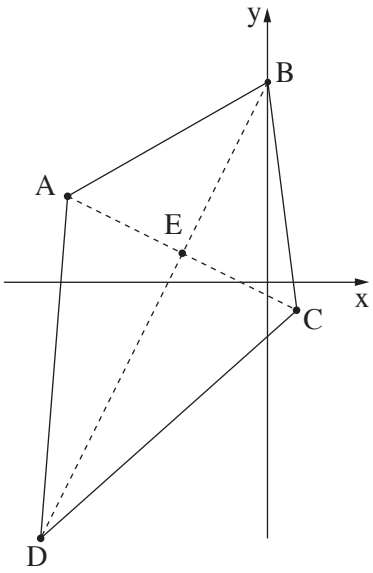
$BA = BC$, $DA = DC$.

The kite's diagonals intersect at Point E .

We know that $A(-7,3)$, $C(1,-1)$,

Vertex B is located on the y-axis, and

Vertex D is located in the third quadrant.



א. **(1)** Find the equation of Diagonal BD .

(2) Find the coordinates of Vertex B .

We know that the area of Kite ABCD is 80 .

ב. Find the length of Diagonal BD . Leave your answer in root form.

ג. Find the coordinates of Vertex D .

ד. Find the equation of the circle whose center is Point A and to which Line BD is tangent. Explain.

3. Some of the inhabitants of a village in China are vegetarians, and all the rest are vegans.

The probability that a villager is a vegetarian is greater by 0.2 than the probability that a villager is a vegan.

80% of the villagers only eat with chopsticks, and the rest only eat with a knife and fork.

25% of the villagers who eat with a knife and fork are vegetarians.

A villager is selected at random.

א. What is the probability that the selected villager is a vegetarian who eats with chopsticks?

ב. **(1)** What is the probability that the selected villager is a vegetarian **or** eats with chopsticks?

(2) We know that the selected villager is a vegetarian **or** eats with chopsticks.

What is the probability that the villager is a vegetarian who eats with chopsticks?

We know that there is a total of 60 villagers living in the village.

ג. Two villagers are selected one after another (without replacement) at random.

What is the probability that the two selected villagers are vegetarians who eat with chopsticks?

Your answer should be accurate to 3 decimal places or written as a simple fraction.

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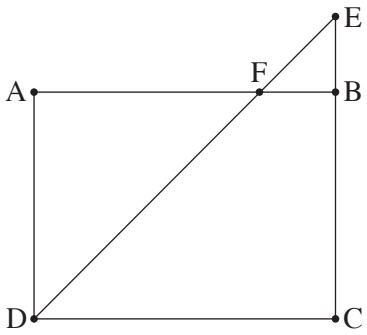
Part Two — Planar Geometry and Planar Trigonometry

4. The quadrilateral ABCD is a rectangle.
Point E is located on the extension of Side CB,
as shown in the figure to the right.
Segment DE intersects Side AB at Point F.



א. Prove that $\Delta AFD \sim \Delta BFE$.

We know that the area of Triangle AFD is 9 times greater than the area of Triangle BFE.

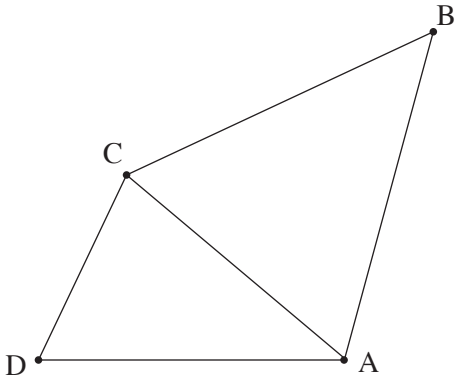
ב. Find the ratio $\frac{AF}{FB}$.

We know that $EB = 4$, and
the area of Rectangle ABCD is 192.

ג. **(1)** Find the length of Side CB.

(2) Find the length of the radius of the circle that circumscribes Triangle DCE.

5. The figure on the right shows two triangles:
Triangle ADC, whose angles are all acute, and
an isosceles triangle ABC, in which $BC = BA$.
We know that the area of Triangle ABC is 113, and
$\sphericalangle ABC = 50^o$.



א. Find the length of Side AB.

ב. Find the length of Side AC.

We know that the radius of the circle that circumscribes Triangle ADC is 8.

ג. Find the size of Angle ADC.

We know that $AD = 15$.

ד. Find the size of Angle CAD.

ה. Find the length of BD.

/Continued on page 5/

Part Three — Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider the function $f(x) = x + \frac{4}{x^2}$.

א. What is the domain [תחום ההגדרה] of function $f(x)$?

ב. Find the coordinates of the intersection point [נקודת החיתוך] of the graph of function $f(x)$ with the x-axis.

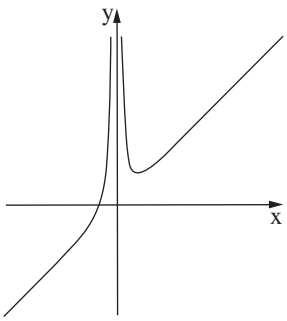
Your answer should be accurate to 2 decimal places.

ג. Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.

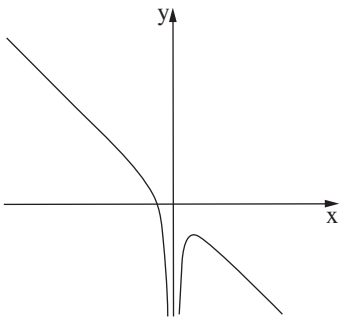
ד. One of the graphs I-III below describes function $f(x)$.

Determine which one it is and explain your answer.

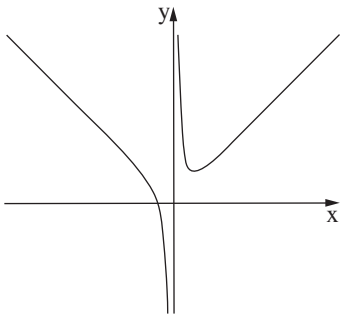
ה. Calculate the area of the region bounded by the graph of function $f(x)$, the line $x = 1$, the line $x = 2$, and the x-axis.



III



II



I

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7. Consider the function $f(x)=1+\frac{1}{2}x-\sqrt{2x+b}$; b is a parameter.

We know that the graph of function $f(x)$ only intersects the x-axis at the point $(8,0)$.

א. Find b .

Substitute $b=9$ and answer Items **ב** to **ד**.

ב. Find the domain [תחום ההגדרה] of function $f(x)$.

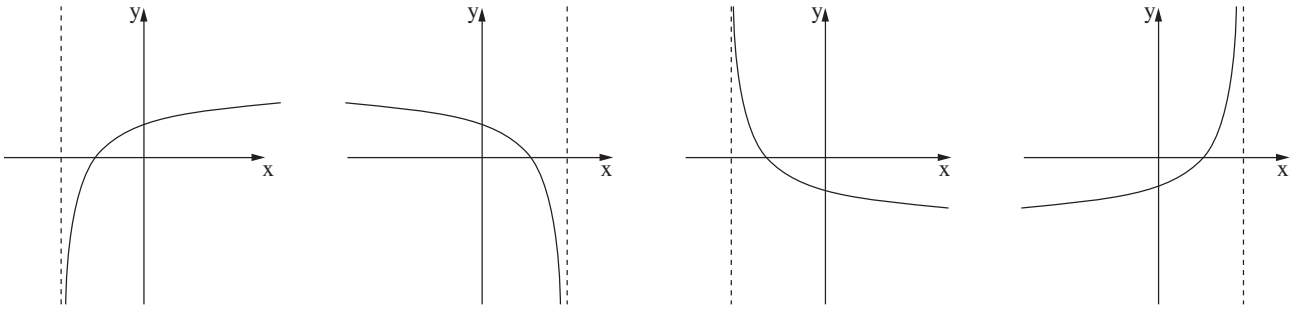
ג. **(1)** Find the coordinates of <u>all</u> the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.

(2) Find the coordinates of the intersection point of the graph of function $f(x)$ with the y-axis.

(3) Sketch a graph of function $f(x)$.

ד. One of the graphs I-IV below describes the derivative function $f'(x)$.

Determine which one it is and explain your answer.



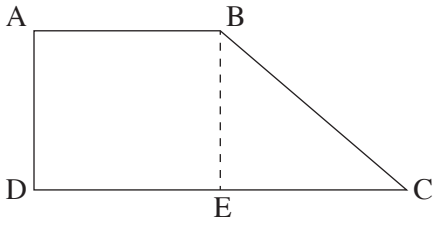
IV **III** **II** **I**

8. Quadrilateral ABCD is a right trapezoid; $AB \parallel DC$ (see figure).

The height of the trapezoid, BE , **bisects** the base DC .

The area of the trapezoid is $12\sqrt{2}$.

Let x denote the length of Side AB .



א. Express the length of the height of the trapezoid in terms of x .

ב. Find the value of x for which the sum of the squares of the legs of the trapezoid (AD^2+BC^2) is minimum (smallest).

ג. Could the sum of the squares of the legs of the trapezoid be 30 ? Explain.

Good Luck!

בהצלחה!