

Mathematics

3 Points — 3rd Exam

מתמטיקה

3 יחידות לימוד — שאלון שלישי

Instructions

א. Duration of the exam: Two hours

הוראות

א. משך הבחינה: שעותיים.

ב. Exam structure and breakdown of points:

ב. מבנה השאלון ומפתח ההערכה:

This exam has six questions on the following topics:

בשאלון זה שש שאלות בנושאים:

Algebra, Differential and Integral Calculus.

אלגברה, חשבון דיפרנציאלי ואינטגרלי.

Answer four questions.

יש לענות על ארבע שאלות – לכל שאלה 25 נקודות.

Each question is worth 25 points.

ג. Material that may be used during the exam:

ג. חומר עזר מותר בשימוש:

(1) A non-graphing calculator. You may not use the programming options of a programmable .

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) Formula sheets (attached)

(2) דפי נוסחאות (מצורפים).

(3) A Hebrew-foreign language/ foreign language-Hebrew dictionary

(3) מילון עברי-לועזי/לועזי-עברי.

ד. Special instructions:

ד. הוראות מיוחדות:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator.

(2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

יש להסביר את כל פעולותיכם, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.

Lack of detail may lower your score or lead to disqualification of your exam.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write in the answer booklet only. Write "טיוטה" at the top of every page you use for rough work.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טיוטה" בראש כל עמוד המשמש טיוטה.

If you use pages outside the answer booklet for rough work, your exam may be disqualified.

כתיבת טיוטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer four of the questions 1-6 (each question – 25 points).

If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Algebra

1. Students at a certain school bought pizzas and cakes for the end-of-year party.

The number of pizzas they bought was 2.5 times greater than the number of cakes.

The students bought a total of 63 pizzas and cakes.

- א. How many cakes did they buy?

The students received a 16% discount on the original price for each cake.

Let x denote the original price of a cake.

- ב. Express the price of a cake after the discount in terms of x .

The price of a pizza is 6 shekels higher than the original price of a cake.

The students paid a total of 3,276 shekels for all the pizzas and cakes they bought.

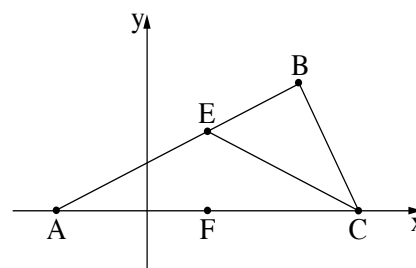
- ג. Find x .

- ד. By what percentage is the price of a pizza greater than the price of a cake after the discount?

2. In Triangle ABC, Vertex A and Vertex C are located on the x-axis, as shown in the figure to the right.

Point E is located on Side AB.

We know that the equation of Line AB is $y = \frac{1}{2}x + 3$
and the y-coordinate of Vertex B is 8.



- א. (1) Find the coordinates of Vertex A.
(2) Find the x-coordinate of Vertex B.

We know that the coordinates of Vertex C are (14, 0).

- ב. Prove that Side AB is perpendicular to Side BC.

We know that the slope of Line EC is $-\frac{1}{2}$.

- ג. (1) Find the equation of Line EC.
(2) Find the coordinates of Point E.
ד. Calculate the area of Triangle EBC.

Point F is located on the x-axis, such that Segment EF is parallel to the y-axis.

- ה. Calculate the area of Quadrilateral FEBC.

3. Consider a circle whose center is located at Point M (2, 0).

Point D (8, -8) is located on the circle (see figure).

- א. (1) Find the radius of the circle.
(2) Write the equation of the circle.

A line with a slope of 1.75 is drawn through Point D.
The line intersects the y-axis at Point E.

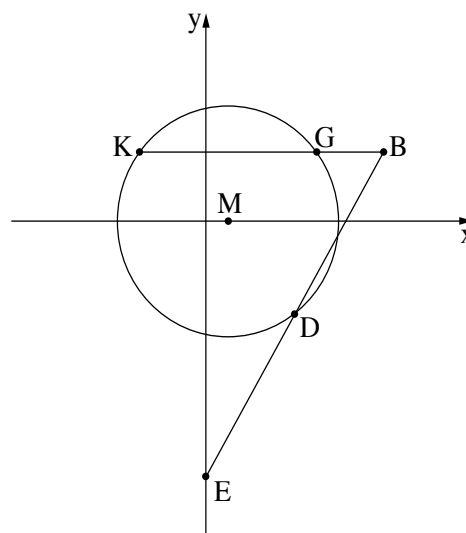
- ב. (1) Find the equation of the line.
(2) Find the coordinates of Point E.

We know that Point B is located on the line, such that
Point D is the midpoint of Segment BE.

- ג. Find the coordinates of Point B.

From Point B, a line was drawn parallel to the x-axis, intersecting the circle at Points G and K, as described in the figure.

- ד. Find the coordinates of Points G and K.



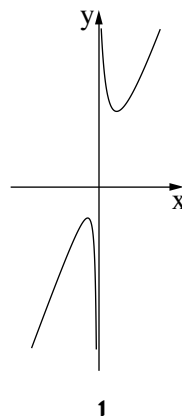
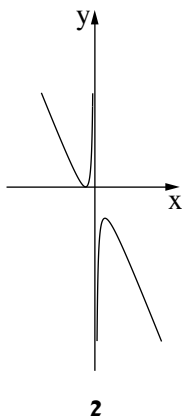
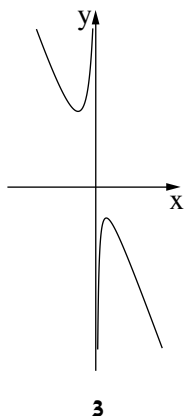
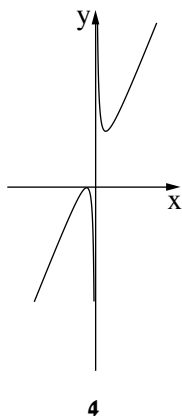
Differential and Integral Calculus

4. Consider the function $f(x) = 8 - 4x - \frac{25}{x}$.

- א. What is the domain [תחום ההגדרה] of function $f(x)$?
- ב. Find the coordinates of the extrema [נקודות קיצון] of function $f(x)$, and determine what type of extrema they are.
- ג. Write the intervals on which function $f(x)$ is decreasing.
- ד. Determine which of the graphs 1–4 at the end of the question describes the graph of function $f(x)$.

A tangent to the graph of function $f(x)$ was drawn through the point at which $x = 5$.

- ה. Find the equation of this tangent.



5. Consider the function $f(x) = x^3 - 12x^2 + 45x - 50$.

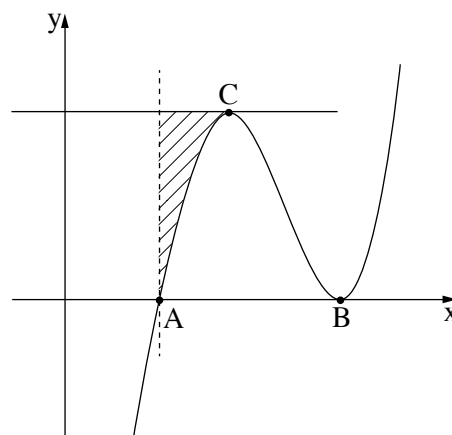
B is the minimum point and C is the maximum point of function $f(x)$.

א. Find the x-coordinate of each of the points B and C.

A tangent to the graph of function $f(x)$ was drawn through Point C.

ב. Find the equation of this tangent.

Point A is one of the intersection points of the graph of function $f(x)$ with the x-axis, as shown in the figure to the right.



ג. Which of the coordinates of the points 1–2 below are the coordinates of Point A? Explain.

1. (1, 0)
2. (2, 0)

A line was drawn through Point A, parallel to the y-axis.

ד. Find the area of the shaded region in the figure:

the area bounded by the graph of function $f(x)$, the tangent, and the line parallel to the y-axis.

6. Consider a rectangular yard, ABCD .

The perimeter of the yard is 140 meters.

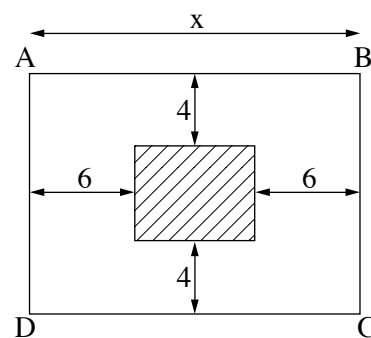
In the center of the yard, a lawn was laid in the form of a rectangle.

The sides of the rectangle of lawn are parallel to the sides of the yard (the shaded area in the figure).

We know that the distance of the lawn from Sides AB and DC is 4 meters from each side, and the distance from Sides BC and AD is 6 meters from each side, as described in the figure.

Let x be the length of Side AB .

- א. Express the length of Side BC in terms of x .
- ב. Express the length of the sides of the lawn in terms of x .
- ג. Find the value of x for which the area of the lawn is greatest (maximal).



Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך