

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Two hours and fifteen minutes

ב. Exam structure and breakdown of points:

There are two parts in this exam, with a total of five questions.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions

Answer three questions of your choice,

at least one question from each part —

$$3 \times 33\frac{1}{3} = 100 \text{ points.}$$

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language/foreign language–Hebrew dictionary.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page.
Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

מתמטיקה

5 יחידות לימוד — שאלון שני

הוראות

א. משך הבחינה: שתיים שעות ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

יש לענות על שלוש שאלות לבחירתכם,

לפחות על שאלה אחת מכל פרק —

$$3 \times 33\frac{1}{3} = 100 \text{ נקודות.}$$

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.

אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי–לועזי/לועזי–עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש.

יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה ומסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.

כתיבת טייטה בדפים שאינם מחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

Questions

Note: Explain all your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer three of the questions 1–5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, Complex Numbers

1. Consider the ellipse whose equation is $\frac{x^2}{9} + \frac{y^2}{b^2} = 1$, b is a positive parameter.

The foci of the ellipse are known to be on the x-axis.

D_1 and D_2 denote the intersection points of the ellipse with the y-axis, F_1 denotes the left focus of the ellipse, and F_2 denotes the right focus.

The quadrilateral $F_1D_1F_2D_2$ is a square.

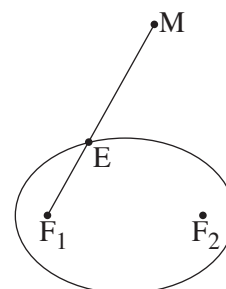
א. (1) Find the value of b .

(2) Calculate the area of the square $F_1D_1F_2D_2$.

Substitute $b^2 = 4.5$ in the ellipse equation, and then answer Items ב–ד.

The point E is a point on the ellipse.

A straight line is drawn from the left focus F_1 through the point E , to a point M (see figure) such that $EM = EF_2$.



- ב. Prove that the locus [המקום הגאומטרי] of all the points M is a circle; write the equation of this circle.

The circle you found in Item ב is shifted to the right by $\frac{3}{\sqrt{2}}$ units, and another circle is obtained. The y-coordinate of each of the points on this other circle is multiplied by $\frac{2}{3}$, and a new curve is obtained.

- ג. Identify the shape of this new curve, and find its equation.

Consider a given triangle: Two of its vertices are the intersection points of the new curve with the x-axis, and its third vertex is also located on the new curve.

- ד. What is the largest possible area of this triangle? Explain your answer.

2. Consider two planes, π_1 and π_2 :

$$\pi_1: (k+2)x + y + (k+1)z + 11 = 0$$

$$\pi_2: (k+1)x + y + z - 5 = 0$$

k is a parameter.

א. Explain why the two planes must intersect.

The two planes intersect at line ℓ_1 ;

this line is known to be parallel to the line $\ell_2: \underline{x} = (1, 2, -1) + m(-1, k, k)$.

ב. (1) Find the value of k .

(2) Find a parametric representation of the line ℓ_1 .

(3) Find the angle between the planes π_1 and π_2 .

Point P is located on the line ℓ_1 and the plane $[yz]$.

The planes π_1 and π_2 intersect the y -axis at Points A and B , respectively.

ג. (1) Find the coordinates of Points P , A , and B .

(2) Find the area of Triangle APB .

3. Consider Equation I : $w^6 = -27$, w is a complex number.

א. Solve Equation I .

Consider Equation II : $\left(z + \frac{\sqrt{3}}{2}i\right)^6 = -27$, z is a complex number.

ב. (1) Use the solutions to Equation I to give the solutions to Equation II in algebraic form.

(2) Explain why the points on the Gauss plane which represent the solutions to Equation II are on a circle; find the equation of this circle.

(3) Prove that all the solutions to Equation II represent vertices of a regular hexagon on the Gauss plane.

Given: The two imaginary solutions and the two real solutions of Equation II represent vertices of a quadrilateral on the Gauss plane.

ג. (1) What type of quadrilateral was obtained? Explain your answer.

(2) Find the ratio between the area of the hexagon and the area of the quadrilateral.

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

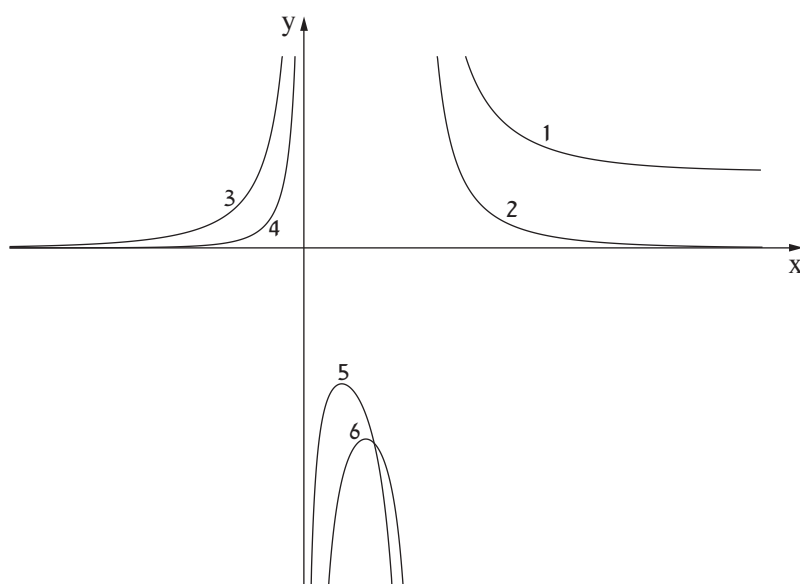
4. Consider the function $f(x) = \frac{2e^{2x}}{e^{2x} - 5e^x + 4}$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
(2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes.
(3) Find the intervals on which function $f(x)$ is increasing and on which it is decreasing.

Consider the function $g(x) = \frac{5e^x}{e^{2x} - 5e^x + 4}$ which is defined on the same domain as function $f(x)$.

- ב. Find the coordinates of the intersection point of the graph of function $f(x)$ and the graph of function $g(x)$.

The figure below shows the graphs of the two functions $f(x)$ and $g(x)$. In the figure, each of the graphs' sections is labeled with a number.



- ג. Identify which function each section marked in the figure belongs to. Explain your answer.
ד. Consider the two expressions I–II below. For each expression, determine whether it is positive or negative. Explain without a calculation.

$$\text{I} \quad \int_{-4}^{-1} (f(x) - g(x)) dx \quad \text{II} \quad \int_{\ln \frac{8}{5}}^{\ln 2} (f(x) - g(x)) dx$$

- ה. Calculate the area of the region bounded by the graphs of the two functions $f(x)$ and $g(x)$ and the lines $x = \ln 9$ and $x = \ln 16$.

5. Consider the function $f(x) = 4x(\ln(x^2) - 1)$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the coordinates of the intersection points of function $f(x)$ with the x-axis.
- (3) Prove that function $f(x)$ is an odd function.
- ב. (1) Find the coordinates of the extrema [נקודות הקיצון] of function $f(x)$, and determine what type of extrema they are.
- (2) Does function $f(x)$ have inflection points [נקודות פיתול]? Explain your answer.
- (3) Sketch a graph of function $f(x)$.

Consider the function $g(x) = \frac{1}{f(x)}$.

- ג. (1) Find the domain [תחום ההגדרה] of function $g(x)$.
- (2) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the axes.
- (3) Sketch a graph of function $g(x)$.
- (4) At how many points do the graphs of functions $f(x)$ and $g(x)$ intersect? Explain your answer.
- ד. Give an example of a primitive function [פונקצייה קדומה] of $g(x)$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך