

## Mathematics

### 4 Points – 1st Exam

#### Instructions

- א. Duration of the exam: Three and a half hours
- ב. Exam structure and breakdown of points:  
This exam consists of three parts, with a total of eight questions.
- Part One – Algebra, Analytical Geometry, Probability  
Part Two – Planar Geometry and Planar Trigonometry  
Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions
- Answer **five** questions of your choice –  $5 \times 20 = 100$  points.
- ג. Material that may be used during the exam:
- (1) A non-graphing calculator. Do not use the programming options of a programmable calculator.  
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached).
- (3) A Hebrew–foreign language / foreign language–Hebrew dictionary.
- ד. Special instructions:
- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each question on a new page. Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.  
Explain all of your work, including calculations, in detail and in a clear and organized fashion.  
Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

Good Luck!

## מתמטיקה

### 4 יחידות לימוד – שאלון ראשון

#### הוראות

- א. משך הבחינה: שלוש שעות וחצי.
- ב. מבנה השאלון ומפתח ההערכה:  
בשאלון זה שלושה פרקים, ובהם שמונה שאלות.
- פרק ראשון – אלגברה, גאומטריה אנליטית, הסתברות  
פרק שני – גאומטריה וטריגונומטריה במישור  
פרק שלישי – חשבון דיפרנציאלי ואינטגרלי של פולינומים, של פונקציות רציונליות ושל פונקציות שורש
- יש לענות על **חמש** שאלות לבחירתכם –  $5 \times 20 = 100$  נקודות.
- ג. חומר עזר מותר בשימוש:
- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.  
שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי–לועזי / לועזי–עברי.
- ד. הוראות מיוחדות:
- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון. יש להסביר את כל פעולותיכם, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.  
חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.  
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

בהצלחה!

## Questions

**Note:** Explain all of your work, including calculations, clearly and in detail.

**Lack of detail may lower your score or lead to the disqualification of your exam.**

Answer five of the questions 1–8 (each question – 20 points).

**Note:** If you answer more than five questions, only the first five answers in your answer booklet will be graded.

### Part One – Algebra, Analytical Geometry, Probability

1. Towns A , B , and C are located on a straight road, as described in the figure below.



The distance between Town B and Town C is 105 km, and the distance between Town B and Town A is 128 km.

A truck and a car left Town B at the same time.

The car traveled at a constant speed in the direction of Town C , and the truck traveled at a constant speed in the direction of Town A .

The speed of the car was 20 km/h higher than the speed of the truck.

The car reached Town C , spent a quarter of an hour in the town, and then headed back to Town B , traveling at the same speed as previously.

When the truck reached Town A , the car was on its way back to Town B and was located 42 km away from Town C .

א. Find the speed of the car and the speed of the truck.

ב. How far was the truck from Town A when the car reached Town C ?

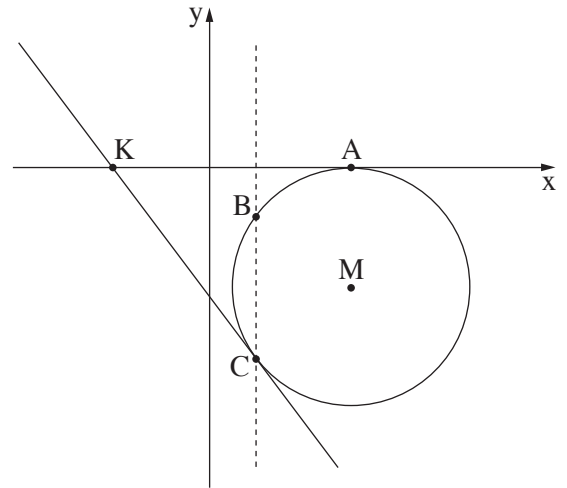
2. The figure to the right shows a circle whose center,  $M$ , is located in the fourth quadrant.

The circle is tangent to the  $x$ -axis at Point  $A(12, 0)$ .

Given: The radius of the circle is 10.

- א. (1) Find the coordinates of the center of the circle,  $M$ .
- (2) Write the equation of the circle.

The circle intersects the line  $x = 4$  at Points  $B$  and  $C$ , as shown in the figure.



- ב. Find the coordinates of Points  $B$  and  $C$ .

The line that is tangent to the circle through Point  $C$  intersects the  $x$ -axis at Point  $K$  (see figure).

- ג. Find the coordinates of Point  $K$ .
- ד. Find the equation of the circle that is tangent to the line  $x = 4$  and whose center is Point  $K$ .
- ה. (1) Find the length of  $KM$ .
- (2) Is the circle whose equation you found for Item ד tangent to the circle shown in the figure (the circle whose center is  $M$ )? Explain your answer.

3. A cash box contains 36 coins: 18 two-shekel coins,  
12 five-shekel coins,  
and 6 ten-shekel coins.

Two coins were randomly removed from the cash box without replacement.

- א. What is the probability that the two coins that were removed were identical?
- ב. We know that the two coins that were removed were identical.  
What is the probability that the sum of the two coins that were removed was greater than 5 shekels?

All the coins were returned to the cash box, and  $x$  ten-shekel coins were added to the cash box.

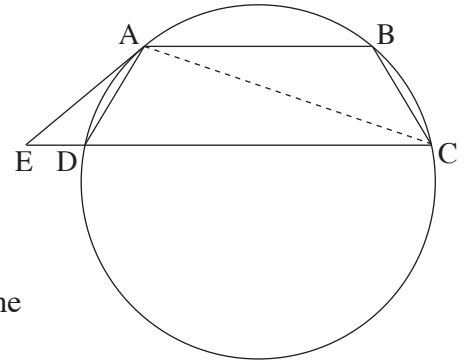
Given: After these coins were added, the probability of randomly removing two five-shekel coins from the cash box without replacement is  $\frac{1}{15}$ .

- ג. Find  $x$ .
- ד. After these coins were added, did the probability of randomly removing two identical coins from the cash box (without replacement) increase, decrease, or remain unchanged? Explain.

## Part Two – Planar Geometry and Planar Trigonometry

4. Quadrilateral ABCD is a trapezoid circumscribed by a circle;  $AB \parallel DC$ .

The tangent to the circle through Point A intersects the extension of Side CD at Point E (see figure).



- א. Prove that ABCD is an isosceles trapezoid.
- ב. Prove that  $\angle ABC = \angle ADE$ .
- ג. Prove that  $\triangle ABC \sim \triangle ADE$ .

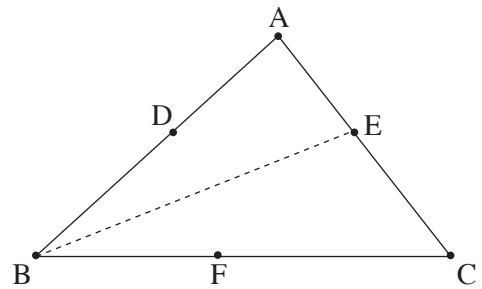
Given: The area of Triangle ABC is 4 times greater than the area of Triangle ADE ;  
 $BC + ED = 15$ .

7. (1) Find the length of Side ED .
- (2) Find the length of Side AB .

5. The figure to the right shows Triangle ABC .

Given:  $BC = 1.5AC$  ;  
 $\angle ACB = 51^\circ$  ;  
the area of Triangle ABC is 21 .

- א. Find the length of Side AC .
- ב. Find the size of Angle ABC .



Given: Point E is located on Side AC , such that BE bisects Angle ABC .

- א. Find the length of BE .

Given: Points D and F are located on Sides AB and BC respectively, such that Quadrilateral BDEF is a rhombus.

7. Find the side length of the rhombus BDEF .

### Part Three – Differential and Integral Calculus of Polynomials, Rational Functions, and Root Functions

6. Consider function  $f(x) = \frac{4x}{x^2 + 4} + a$ ;  $a$  is a parameter.

- א. Find the domain [תחום ההגדרה] of function  $f(x)$ .
- ב. Find the coordinates of the extrema [נקודות הקיצון] of function  $f(x)$ , and determine what type of extrema they are (express in terms of  $a$ , if necessary).

Given: The minimum point of function  $f(x)$  is located on the  $x$ -axis.

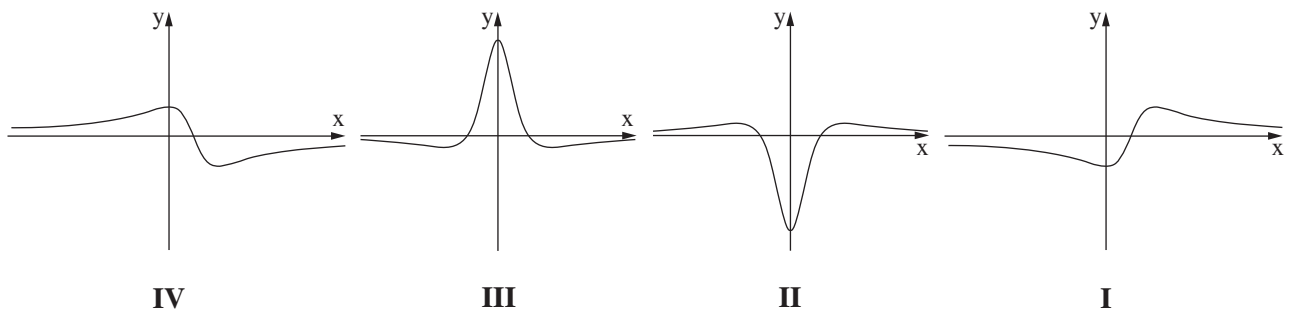
- ג. Find  $a$ .

Substitute  $a$ , which you found in Item ג, in function  $f(x)$  and answer Items ד–ו.

- ד. Find the equations of the asymptotes of function  $f(x)$  that are perpendicular to the axes (if there are any).
- ה. Sketch a graph of function  $f(x)$ .

Consider function  $g(x) = 3 \cdot f(x)$ .

- ו. (1) One of the graphs I–IV below describes the derivative function  $g'(x)$ . Determine which one it is, and explain your answer.
- (2) Find the area of the region bounded by the graph of the derivative function  $g'(x)$ , the line  $x = 1$ , and the axes.



7. Consider function  $f(x) = x^2 \cdot \sqrt{4x + 20}$ .

- א. Find the domain [תחום ההגדרה] of function  $f(x)$ .
- ב. Find the coordinates of the intersection points [נקודות החיתוך] of the graph of function  $f(x)$  with the axes.
- ג. Find the coordinates of all the extrema [נקודות הקיצון] of function  $f(x)$ , and determine what type of extrema they are.
- ד. Sketch a graph of function  $f(x)$ .

Consider function  $g(x) = f(x) + c$ ;  $c$  is a parameter.

Given: The line  $y = 12$  is tangent to the graph of function  $g(x)$ .

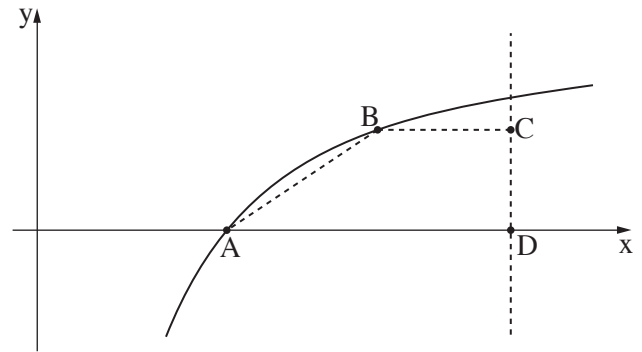
- ה. Find  $c$  (give both possibilities).

8. The figure to the right shows part of the graph of function  $f(x) = 1 - \frac{2}{x}$  over the interval  $x > 0$ .

The graph of function  $f(x)$  intersects the  $x$ -axis at Point A.

Point B is located on the graph of function  $f(x)$  in the first quadrant, to the left of the line  $x = 5$ .

A line is drawn from Point B, parallel to the  $x$ -axis and intersecting the line  $x = 5$  at Point C.



Given:  $D(5, 0)$

- א. Find the coordinates of Point A.

Let  $t$  be the  $x$ -coordinate of Point B.

- ב. Express the coordinates of Points B and C in terms of  $t$ .
- ג. Find the coordinates of Point B for which the area of Trapezoid ABCD is maximum (greatest).
- ד. Show that the maximum area of Trapezoid ABCD is 1.

**Good Luck!**

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**בהצלחה!**

זכות היוצרים שמורה למדינת ישראל  
אין להעתיק או לפרסם אלא ברשות משרד החינוך