

Mathematics

3 Points – 3rd Exam

מתמטיקה

3 יחידות לימוד – שאלון שלישי

Instructions

א. Duration of the exam: Two hours

הוראות

א. משך הבחינה: שעותיים.

ב. Exam structure and breakdown of points:

This exam has six questions on the following topics:
Algebra, Differential and Integral Calculus.
Answer four questions – each question is worth
28 points.
The maximum number of points you can earn
is 100.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שש שאלות בנושאים:
אלגברה, חשבון דיפרנציאלי ואינטגרלי.
יש לענות על ארבע שאלות – לכל שאלה 28 נקודות.
סך הנקודות לא יעלה על 100.

ג. Material that may be used during the exam:

- (1) A non-graphing calculator. You may not use the programming options of a programmable calculator.
Use of a graphing calculator or programming options may lead to the disqualification of your exam.
- (2) Formula sheets (attached)
- (3) A Hebrew–foreign language/foreign language–Hebrew dictionary

ג. חומר עזר מותר בשימוש:

- (1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות. שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.
- (2) דפי נוסחאות (מצורפים).
- (3) מילון עברי–לועזי/לועזי–עברי.

ד. Special instructions:

- (1) Do not copy the question, but do indicate the number of the question you are answering.
- (2) Start each answer on a new page. Write the steps of the solution in your answer booklet, even when the calculations are done with a calculator. Explain all your work, including calculations, in detail and in a clear and organized fashion. Lack of detail may lower your score or lead to disqualification of your exam.

ד. הוראות מיוחדות:

- (1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.
- (2) יש להתחיל כל שאלה בעמוד חדש. יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.
- יש להסביר את כל פעולותיכם, כולל חישובים, בפירוט ובצורה ברורה ומסודרת.
- חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

Write in the answer booklet only. Write "טייטה" at the top of every page you use for rough work.
If you use pages outside the answer booklet for rough work, your exam may be disqualified.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה.
כתיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Answer four of the questions 1–6 (each question – 28 points).

If you answer more than four questions, only the first four answers in your answer booklet will be graded.

Note: Explain all your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to disqualification of your exam.

Algebra

1. The price of a printed T-shirt at a certain store was 18% higher than the price of a plain T-shirt. The total cost of 4 printed T-shirts and 3 plain T-shirts at this store was 386 shekels.
- א. Find the price of one printed T-shirt and the price of one plain T-shirt.

The store had a few printed T-shirts and a large number of plain T-shirts left in stock, so they changed the prices of the T-shirts.

The price of a printed T-shirt went up by 7 shekels, and the price of a plain T-shirt was reduced by 16% .

- ב. Find the price of one printed T-shirt and the price of one plain T-shirt, after the change in prices.

After the change in prices, a class of 11th grade students ordered T-shirts from the store.

They ordered 12 more printed T-shirts than plain T-shirts.

They paid a total of 5,760 shekels for all of the T-shirts they ordered.

- ג. How many plain T-shirts did the 11th grade students order?

2. The figure to the right shows a right triangle ABD ($\angle ABD = 90^\circ$).

Vertex A is located on the x-axis,

and Vertex B is located on the y-axis.

The equation of Side AB is $y = \frac{1}{4}x + 2$.

א. Find the coordinates of Vertex A and Vertex B.

ב. Find the equation of Side BD.

The x-coordinate of Vertex D is -2 .

ג. (1) Find the y-coordinate of Vertex D.

(2) Prove that Triangle ABD is an isosceles triangle.

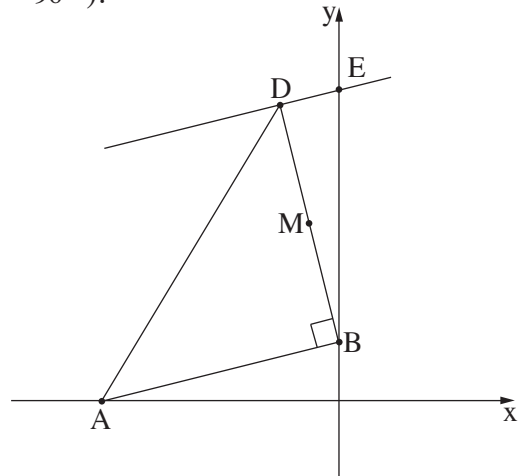
A line was drawn through Vertex D, parallel to Side AB.

This line intersects the y-axis at Point E.

ד. Find the coordinates of Point E.

Point M is the midpoint of Segment DB.

ה. Find the area of Triangle BME.



3. A circle whose center is M intersects the y -axis at Points A and D , as shown in the figure to the right.

Each of the lines DB and AC is a diameter in the circle.

The equation of Line AC is $y = \frac{4}{3}x + 3$,
and the equation of Line DB is $y = -\frac{4}{3}x + 11$.

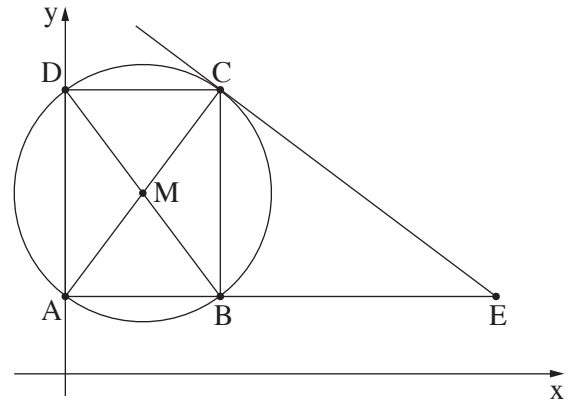
- א. Find the coordinates of Points A and D .
- ב. Find the coordinates of Point M .
- ג. (1) Find the length of the radius of the circle.
(2) Write the equation of the circle.
- ד. Find the coordinates of Point C .

A tangent to the circle was drawn through Point C .

- ה. Find the equation of this tangent.

Point E is located on the tangent, such that Segment AE is parallel to the x -axis.

- ו. (1) Find the coordinates of Point E .
- (2) Find the area of Trapezoid $ADCE$.



Differential and Integral Calculus

4. Consider function $f(x) = 3x - 12\sqrt{x} + 15$.

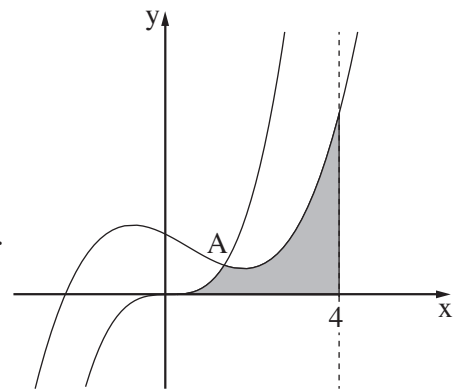
- א. What is the domain [תחום ההגדרה] of function $f(x)$?
- ב. Find the coordinates of the intersection point of the graph of function $f(x)$ with the y-axis.
- ג. (1) Find the coordinates of the internal extremum [נקודת הקיצון הפנימית] of function $f(x)$, and determine what type of extremum it is.
(2) Find the interval on which function $f(x)$ is increasing.
- ד. Which one of the statements 1–3 below is correct? Explain your answer.
 1. Function $f(x)$ intersects the x-axis at only one point.
 2. Function $f(x)$ intersects the x-axis at two points.
 3. Function $f(x)$ does not intersect the x-axis.

5. The figure to the right shows the graphs of two functions:

$$f(x) = 2x^3 - 12x + 18,$$

$$g(x) = 2x^3.$$

- א. Determine which graph passes through the origin: the graph of function $f(x)$ or the graph of function $g(x)$. Explain.
- ב. Find the coordinates of Point A, which is the intersection point of the graphs of the two functions.
- ג. Calculate the area of the shaded region in the figure: the region bounded by the graph of function $f(x)$, the graph of function $g(x)$, the x-axis, and the line $x = 4$.



6. Consider function $f(x) = -2x^2 + 9x$, and the line whose equation is $y = 3x$.

Point A is located on the graph of function $f(x)$ in the first quadrant, above the line.

Point B is located on the line, such that Segment AB is parallel to the y-axis.

Point C is located on the y-axis, such that Segment AC is perpendicular to AB (see figure).

Let x be the x-coordinate of Point A.

- א. (1) Express the coordinates of Points A and B in terms of x .
- (2) Express the length of Segment AB in terms of x .
- (3) Express the area of Triangle ABC in terms of x .
- ב. Find the value of x for which the area of Triangle ABC is maximum (greatest).

