

Note: There are special instructions in this exam.

Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.
יש לענות על השאלות על פי הנחיות אלה!

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

There are two parts in this exam, with a total of five questions.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions

Answer three questions of your choice, at least one from each part
— $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator.

Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.

Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

הוראות

א. משך הבחינה: שתיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

יש לענות על שלוש שאלות לבחירתכם,

לפחות שאלה אחת מכל פרק

— $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי. אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול

לגרם לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש.

יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה מסודרת.

חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה. תיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer three of the questions 1-5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers

1. Consider the points $A(-5, 3)$ and $B(0, -2)$.

א. Find the equation of the locus (מקום גאומטרי) of the centers of the circles in which Segment AB is a chord.

Circle M is one of the circles in which Segment AB is a chord.

The intersection points of Circle M with the x -axis are the foci of an ellipse whose equation is in canonical form.

ב. Find the coordinates of the center of Circle M and the radius of Circle M .

Given: The length of the major axis of the ellipse is equal to the length of the diameter of Circle M .

ג. What is the equation of the ellipse?

Let F be the right focus of the ellipse. A line that is perpendicular to the x -axis passes through the left focus of the ellipse. This perpendicular line intersects the ellipse at Points Q and T , and the circle M at Points K and L .

ד. Find the ratio between the area of Triangle KLF and the area of Triangle TQF .

2. Consider Pyramid OABC whose base is a triangle ABC .

Let: $\vec{OA} = \underline{u}$, $\vec{OB} = \underline{v}$, $\vec{OC} = \underline{w}$.

Given: $\angle AOB = \angle BOC = \angle COA = 90^\circ$, $|\underline{w}| = |\underline{v}| = |\underline{u}|$.

Point H satisfies $\vec{OH} = t\underline{u} + s\underline{v} + k\underline{w}$. t , s , and k are parameters.

Given: $\vec{OH}$ is perpendicular to the pyramid base ABC .

א. Prove that $t = s = k$.

Point M is located on the pyramid base ABC and is the intersection of the medians of the base (centroid of the base).

ב. Prove that $\vec{OM} = \frac{1}{3}\underline{u} + \frac{1}{3}\underline{v} + \frac{1}{3}\underline{w}$, and explain why OM is the vertical height of the pyramid (height to the pyramid base ABC) .

The height to the base ABC rests on the line ℓ ; Point P is located on the line ℓ .

ג. Express in terms of $\underline{w}$, $\underline{v}$, $\underline{u}$ the vector $\vec{OP}$ for which the volume of Pyramid PABC is double the volume of Pyramid OABC (two possibilities).

Pyramid OABC is placed in a coordinate system. Point O is at the origin, Point A is on the positive x-axis, Point B is on the positive y-axis, and Point C is on the positive z-axis.

Given: $|\underline{u}| = a$.

ד. Find the vector equation (הצגה פרמטרית) of the line ℓ on which the segment OP lies.

ה. Express the equation of the plane ABC in terms of a .

ו. The given volume of Pyramid OABC is $20\frac{5}{6}$. Calculate a .

3. The number $z = R(\cos \alpha + i \cdot \sin \alpha)$ is located in the third quadrant on the Gauss plane.

Given: $\frac{z}{\bar{z}} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

א. Find α .

Given: $|2iz| + \left|\frac{\bar{z}}{i}\right| - \left|\frac{z}{\bar{z}}\right| = 8$.

ב. Find R .

ג. Consider the given equation $w^9 = \frac{z^3}{27}$ (z is the number you found).

Show that the number $\frac{z}{\bar{z}}$ is one of the solutions of this equation.

ד. Triangle ABC is an isosceles triangle (משולש שווה שוקיים). Its base vertices, B and C, correspond to the numbers $\frac{z}{\bar{z}}$ and $\frac{\bar{z}}{z}$.

The apex A corresponds to the number $z + k$; k is a purely imaginary number.

(1) What is the value of k ?

(2) Calculate the area of the quadrilateral ABOC (Point O is the origin).

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the given function: $f(x) = x^2 e^{a-x^3}$, which is defined for all x ; a is a parameter.
- א. (1) Find the interval on which function $f(x)$ is positive.
- (2) Find the x -coordinates of the extrema (נקודות קיצון) of function $f(x)$, and determine what type of extrema they are.

The area of the region bounded by the graph of the derivative function $f'(x)$ and the x -axis is $3\sqrt{\frac{4e}{9}}$.

- ב. Find the value of a .

Substitute $a = 1$ and answer items ג-ה.

- ג. Sketch a graph of function $f(x)$.

Function $f(x)$ is the derivative of function $g(x)$ ($g'(x) = f(x)$).

- ד. (1) On what interval is function $g(x)$ increasing? Explain.
- (2) How many inflection points (נקודות פיתול) does function $g(x)$ have? Explain.

Let B be the inflection point at which the value of function $g(x)$ is the highest of all its inflection points.

Given: The y -coordinate of Point B is $\frac{e - \sqrt[3]{e}}{3}$.

- ה. Find function $g(x)$.

5. Consider a given function $f(x)$ which satisfies the following properties: the function is defined for all x and is continuous; the function is odd; the line $y = 0$ is an asymptote of the function; and the function has a single minimum whose coordinates are $(-1, -a)$, a is a positive parameter.
- א. Sketch a possible graph of function $f(x)$.

Consider the given function $h(x) = \ln(f(x))$.

- ב. (1) Find the domain (תחום ההגדרה) of the function $h(x)$.
- (2) Find the equations of the asymptotes of function $h(x)$ that are perpendicular to the axes (if there are any).
- (3) Find the range of values of a for which the graph of function $h(x)$ intersects the x -axis at two points.
- (4) Sketch a graph of function $h(x)$, if we know that its graph intersects the x -axis at two points.

Given: $f(x) = \frac{4x}{1+x^2}$.

$g(x)$ is a function that satisfies both $g'(x) = f(x)$ and $g(0) = 0$.

- ג. (1) Find function $g(x)$.
- (2) Is function $g(x)$ even, odd, or neither even nor odd? Explain.

Consider the integral $\int_{-5}^t g(x) dx$, $t > -5$.

- ד. What value of t satisfies $2 \cdot \int_{-5}^t g(x) dx = \int_{-5}^5 g(x) dx$? Explain.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך