

Note: There are special instructions in this exam.

Answer the questions according to these instructions.

שימו לב: בבחינה זו יש הנחיות מיוחדות.

יש לענות על השאלות על פי הנחיות אלה.

Mathematics

5 Points – 2nd Exam

Instructions

א. Duration of the exam: Two hours and fifteen minutes.

ב. Exam structure and breakdown of points:

There are two parts in this exam, with a total of five questions.

Part One: Analytical Geometry, Vectors,
Spatial Trigonometry, Complex Numbers

Part Two: Growth and Decay, Power Functions,
Exponential and Logarithmic Functions

Answer three questions of your choice, at least one from each part
— $3 \times 33\frac{1}{3} = 100$ points.

ג. Material that may be used during the exam:

(1) A non-graphing calculator. You may not use the programming options of a programmable calculator. Use of a graphing calculator or programming options may lead to the disqualification of your exam.

(2) Formula sheets (attached).

(3) A Hebrew-foreign language/foreign language-Hebrew dictionary.

ד. Special instructions:

(1) Do not copy the question, but do indicate the number of the question you are answering.

(2) Start each question on a new page.

Write the steps of the solution in the answer booklet, even when the calculations are done with a calculator.

Explain all of your work, including calculations, in detail and in a clear and organized fashion.

Lack of detail may lower your score or lead to the disqualification of your exam.

Write only in the answer booklet. Write "טייטה" at the top of each draft page. If you write draft material on paper outside the answer booklet, your exam may be disqualified.

הוראות

א. משך הבחינה: שעותיים ורבע.

ב. מבנה השאלון ומפתח ההערכה:

בשאלון זה שני פרקים, ובהם חמש שאלות.

פרק ראשון: גאומטריה אנליטית, וקטורים,

טריגונומטריה במרחב, מספרים מרוכבים

פרק שני: גדילה ודעיכה, פונקציות חזקה,

פונקציות מעריכיות ולוגריתמיות

יש לענות על שלוש שאלות לבחירתכם,

לפחות שאלה אחת מכל פרק

— $3 \times 33\frac{1}{3} = 100$ נקודות.

ג. חומר עזר מותר בשימוש:

(1) מחשבון לא גרפי.

אין להשתמש באפשרויות התכנות במחשבון שיש בו אפשרות תכנות.

שימוש במחשבון גרפי או באפשרויות התכנות במחשבון עלול לגרום לפסילת הבחינה.

(2) דפי נוסחאות (מצורפים).

(3) מילון עברי-לועזי/לועזי-עברי.

ד. הוראות מיוחדות:

(1) אין להעתיק את השאלה; יש לסמן את מספרה בלבד.

(2) יש להתחיל כל שאלה בעמוד חדש.

יש לרשום במחברת את שלבי הפתרון, גם כאשר החישובים מתבצעים בעזרת מחשבון.

יש להסביר את כל הפעולות, כולל חישובים, בפירוט ובצורה ברורה מסודרת. חוסר פירוט עלול לגרום לפגיעה בציון או לפסילת הבחינה.

יש לכתוב במחברת הבחינה בלבד. יש לרשום "טייטה" בראש כל עמוד המשמש טייטה. תיבת טייטה בדפים שאינם במחברת הבחינה עלולה לגרום לפסילת הבחינה.

Good Luck!

בהצלחה!

Questions

Note: Explain all of your work, including calculations, clearly and in detail.

Lack of detail may lower your score or lead to the disqualification of your exam.

Answer three of the questions 1-5, at least one question from each part (each question – $33\frac{1}{3}$ points).

Note: If you answer more than three questions, only the first three answers in your answer booklet will be graded.

Part One – Analytical Geometry, Vectors, Spatial Trigonometry, and Complex Numbers

1. Consider two given circles that are externally tangent to each other.

The center of one circle, whose equation is $(x - a)^2 + y^2 = r^2$, is Point M ,

a is a positive parameter.

The center of the other circle, whose equation is $(x - 13)^2 + y^2 = R^2$, is Point N .

The length of the segment connecting the centers of the two circles is 9, and the ratio between the lengths of the circles' radii is 1:2, $r < R$.

- א. Find the equation of the circle whose center is N and both possible equations for the circle whose center is M .

Given: $a < 13$.

- ב. On one set of axes, draw a sketch of the two circles and all the tangents that are common to both circles.
- ג. Find the equation of the tangent that passes through the point that is common to both circles.
- ד. The line $mx - y + n = 0$ is a common tangent to both circles. Find m and n (two possibilities).

Consider two other given circles that are externally tangent to each other.

The equations of the circles are: $(x - t)^2 + y^2 = r^2$; $(x - k)^2 + y^2 = R^2$; t and k are parameters.

The values of the radii r and R are the same as those you found in Item א.

- ה. Could the two lines whose equations you found in Item ד also be tangent to these circles? If they could, find t and k . If not, explain.

2. Consider four given points that are located on the same plane:

$A(4, p, -1)$, $B(7, 5, 5)$, $C(1, -1, 2)$, $D(-2, 5, -4)$, p is a parameter.

- א. Find the equation of the plane $ABCD$.
- ב. Calculate the value of the parameter p .
- ג. Prove that the quadrilateral $ABCD$ is a square.

Point S is a vertex of a pyramid $SABCD$ whose base is $ABCD$.

The edge SC lies on the line $\underline{x} = (0, -4, 1) + t(1, 3, 1)$.

Given: The volume of the pyramid is 81 .

- ד. Find the coordinates of Point S (two possibilities).

Consider an additional plane π that is perpendicular to the edge SC .

- ה. Find the angle between the plane $ABCD$ and the plane π .

3. Consider the given equation: $z^2 + z\bar{z} = z + 2\bar{z} + 9 + 7i$, z is a complex number.

z_1 is one of the solutions to the equation, and it represents a point located on the Gauss plane in the first quadrant, on a circle whose center is at the origin.

- א. Find the equation of the circle.

The circle circumscribes a square. z_1 represents one of the vertices of this square.

- ב. Calculate the area of the square.
- ג. Find the coordinates of the square's other vertices.

Each of the numbers representing the two vertices of the square in the first and third quadrants is multiplied by $r_1 \cdot (\cos(\alpha) + i \sin(\alpha))$, and each of the numbers representing the two vertices of the square in the second and fourth quadrants is multiplied by $r_2 \cdot (\cos(\alpha + 30^\circ) + i \sin(\alpha + 30^\circ))$.

The points on the Gauss plane representing the results obtained from this multiplication form a new convex quadrilateral on the Gauss plane.

Given: r_1 and r_2 are positive , $r_1 \neq r_2$.

- ד. What type of quadrilateral is obtained? Explain your answer.

Given: The area of the quadrilateral obtained is $\sqrt{3}$ times greater than the area of the square you calculated in Item ב.

- ה. Calculate $r_1 \cdot r_2$.

Part Two — Growth and Decay, Power Functions, Exponential and Logarithmic Functions

4. Consider the given function: $f(x) = xe^x - 2e^x + 1$, which is defined for all x .
- א. (1) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the y-axis (if there are any).
- (2) Find the coordinates of the intersection point of the graph of function $f(x)$ with the y-axis.
- (3) Find the intervals on which function $f(x)$ is increasing and the intervals on which it is decreasing.
- (4) Sketch a graph of function $f(x)$.

Consider the given function: $g(x) = \frac{1 - e^x}{e^x - x}$, which is defined for all x .

- ב. (1) Find the equations of the asymptotes of function $g(x)$ that are perpendicular to the y-axis.
- (2) Find the coordinates of the intersection points of the graph of function $g(x)$ with the x-axis (if there are any).
- (3) Prove that $g'(x) = \frac{f(x)}{(e^x - x)^2}$.
- ג. Use the sketch of the graph of function $f(x)$ to find out how many maxima and how many minima function $g(x)$ has. Explain your answer.
- ד. Sketch a graph of function $g(x)$.
- ה. Calculate the area of the region bounded by the graph of function $g(x)$, the x-axis, and the line $x = -1$.

5. Consider the given function: $f(x) = x + \ln(x^2 - 8)$.

- א. (1) Find the domain [תחום ההגדרה] of function $f(x)$.
- (2) Find the equations of the asymptotes of function $f(x)$ that are perpendicular to the axes (if there are any).
- (3) Find the coordinates of the extremum [נקודת הקיצון] of function $f(x)$, and determine what type of extremum it is.
- (4) Sketch a graph of function $f(x)$.

$f'(x)$ is the derivative function of function $f(x)$.

- ב. (1) Find the domain of the derivative function $f'(x)$.
- (2) Find the equations of the asymptotes of the derivative function $f'(x)$ that are perpendicular to the axes.
- (3) Find the coordinates of the intersection points of the derivative function $f'(x)$ with the axes (if there are any).
- (4) Sketch a graph of the derivative function $f'(x)$, if we know that it has no extrema.

Consider the given function: $g(x) = e^{f(x)}$ that is defined on the same domain as function $f(x)$.

- ג. (1) Find the coordinates of the extremum of function $g(x)$, and determine what type of extremum it is.
- (2) Find the intervals on which function $g(x)$ is increasing and the intervals on which it is decreasing.
- ד. Calculate the area of the region bounded by the function $y = f'(x) \cdot g(x)$, the x-axis, and the lines $x = -4$ and $x = -5$.

Good Luck!

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בהצלחה!

זכות היוצרים שמורה למדינת ישראל
אין להעתיק או לפרסם אלא ברשות משרד החינוך